NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 1204

FACULTY OF APPLIED SCIENCE

DEPARTMENT OF APPLIED MATHEMATICS

SMA1204: ORDINARY DIFFERENTIAL EQUATIONS

MARCH 2024 EXAMINATION:

Time : 3 hours

Candidates should attempt **ALL** questions from Section **A** [**40 marks**] and **ANY THREE** Questions in Section **B** [**60 marks**].

SECTION A: Answer ALL questions in this section [40]

A1. Give detailed explanations for the following phrases:

(a) An ordinary differential equation. [2]

(b) The solution of a differential equation. [2]

(c) An inexact ordinary differential equation. [2]

(d) An integrating factor. [2]

(e) A normal differential equation. [2]

A2. (a) Solve the differential equation $\sin^{-1}(\frac{dy}{dx}) + y = x$ by using the substitution $v = x - y$. [5]

(b) Use separation of variables to solve

$$\ln x \; dy = \frac{1 + e^y}{x} dx.$$

[5]

(b) Find the critical points of the system

$$x' = x^2y + 3xy - 10y,$$
$$y' = xy - 4x.$$

[6]

(c) Characterize each of the critical ponts and hence check their stability. [8]

B7. (a) Use variation of parameters to obtain derivatives of the two varying parameters in the particular solution of the differential equation $-y'' = \sec t$. [10]

(b) Show that

$$\phi(x) = e^{x^2} \int_{x_0}^{x} e^{-t^2}\,dt + e^{x^2}$$

is a solution of any first-order differential equation $y' = 2xy + 1$. [5]

(c) Use a suitable substitution to show that the independent variable in the differential equation

$$\frac{d\nu}{du} = \Gamma(a_1u + a_2\nu + a_3),$$

a_1, a_2, a_3 constants, is given by

$$\int \frac{du}{a_1 + a_2\Gamma(u)} + A.$$

[5]

B8. The system of differential equations below describes a mutual interaction between two species N_1 and N_2.

$$\frac{dN_1}{dt} = r_1N_1(1 - \frac{N_1}{k_1} + b_{12}\frac{N_2}{k_1})$$
$$\frac{dN_2}{dt} = r_2N_2(1 - \frac{N_2}{k_2} + b_{21}\frac{N_1}{k_2}).$$

Let $x_1 = \frac{N_1}{k_1}$, $x_2 = \frac{N_2}{k_2}$, $\tau = r_1t$, $\rho = \frac{r_2}{r_1}$, $a_{12} = \frac{b_{12}k_2}{k_1}$ and $a_{21} = \frac{b_{21}k_1}{k_2}$.

(a) Show that the τ derivative of x_1 is $x_1(1 - x_1 + a_{12}x_2)$. [5]

(b) Obtain an expression for the τ derivative of x_2. [4]

(c) Obtain the four equilibrium points of the new system. [7]

(d) Calculate the Jacobian of the new system in terms of x_1 and x_2. [4]

END OF QUESTION PAPER