NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCE

DEPARTMENT OF APPLIED MATHEMATICS

SMA1204: ORDINARY DIFFERENTIAL EQUATIONS

MARCH 2025 EXAMINATION:

Time : 3 hours

Candidates should attempt **ALL** questions from Section **A** **[40 marks]** and **ANY THREE** Questions in Section **B** **[60 marks]**.

SECTION A: Answer ALL questions in this section [40]

A1. Use separation of variables to solve the following:

(a)

$$\frac{dy}{dx} = \frac{\cos 2x}{1 + 4\sin x \cos x}.$$

[5]

(b) $(1 + y^2)dx = (1 + e^x)dy.$ [5]

A2. Given the differential equation

$$(1 + x^2)\frac{dy}{dx} + 4xy = \frac{4x}{(1 + x^2)^3}.$$

(a) Show that the differential equation is not exact. [3]

(b) Obtain the integrating factor $\mu(x) = e^{\int p(x)dx}$ where $p(x) = \frac{4x}{1+x^2}$. [4]

(c) Hence, solve the differential equation. [8]

A3. Solve the homogeneous differential equation $y^{viii} + 8y^{iv} + 16y = 0$. [6]

A4. (a) Show that the solution $\eta(x, y) = c$ of an exact differential equation

$$P(x, y)dx + Q(x, y)dy = 0$$

is such that $\eta_x = P(x, y)$ and $\eta_y = Q(x, y)$. [4]

(b) Given the differential equation

$$e^{(\ln x^2 + \sin t)} \cos t dt + 2xe^{\sin t} dx = 2t dt,$$

find $\eta(x, t)$ such that $d\eta(x, t) = P(x, t)dx + Q(x, t)dt$. [5]

SECTION B: Answer ANY THREE questions in this section [60]

B5. (a) Given a general second-order non-homogeneous differential equation with variable coefficients $a_2(x)y'' + a_1(x)y' + a_0(x)y = f(x)$. Outline the steps that you would take to solve this equation by variation of parameters. [7]

(b) Hence solve the second order differential equation $-y'' = \sec t$. [7]

(c) Use the Method of Undetermined coefficients to solve the differential equation $y^{iv} + y''' = 1 - x^2 e^{-x}$. [6]

B6. (a) Determine the critical points of the system

$$x' = 10x - 5xy,$$

$$y' = 3y + xy - 3y^2.$$

[5]

(b) Find the Jacobian of the system and evaluate it at each of the critical points. Hence determine the stability of the critical points. [5]

(c) Solve the differential equation $\sqrt{(1 - x^2)}\, dy = \sqrt{\sin^{-1}(x)}\, dx$. [10]

B7. (a) A water tank has vertical sides and a horizontal rectangular base. The area of the base is 2 m^2. At time $t = 0$, the tank is empty and water begins to flow into the tank at a rate of 1 m^3 per hour. At the same time water begins to flow out from the base at a rate of $0.2\sqrt{h}\ m^3$ per hour, where h is the depth of water in the tank at time t hours.

Form a differential equation satisfied by h and t and show that the time T hours taken for the depth of water to reach 4m is given by

$$T = \int_0^4 \frac{10}{5 - \sqrt{h}} dh.$$

[10]

(b) Use the substitution $v = x - y$ to solve $\cos^{-1}(y') = x - y$. [5]

(c) Solve the differential equation

$$\frac{dy}{d\theta} = \sin 2\theta\sqrt{1 + 2\cos^2\theta}.$$

[5]

B8. (a) Show that $xy' + y = 2$ is exact and solve it. [6]

(b) Solve the same differential equation $xy' + y = 2$ by using an integrating factor. [6]

(c) Use separation of variables to solve

$$\frac{dy}{dx} = \frac{\sin y}{1 + \cos 2x}.$$

[8]

END OF QUESTION PAPER