NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA1204

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA1204:ORDINARY DIFFERENTIAL EQUATIONS

OCTOBER 2024 EXAMINATION

Time : 3 hours

Candidates should attempt all questions from Section A [**40 MARKS**] and ANY THREE questions in Section B [**60 MARKS**].

SECTION A: Answer ALL questions in this section [40].

A1. Explain the following phrases, giving examples where necessary:

(a) an exact differential equation, [3]

(b) an integrating factor, [3]

(c) a homogenous solution. [3]

A2. Determine the order of each of the following ordinary differential equations

(a) $x^4\frac{dy}{dx} + \frac{d^5y}{dx^5} = y + x^7$ [1]

(b) $y''' + cosy'' + x^2y' = x^3$ [1]

(c) $(x')^2x''' = x^4x'' + t^5x'$ [2]

A3.

Determine whether the given function $y = \phi(x)$ is a solution of the given ordinary differential equation.

(a) $\phi(x) = \dfrac{1-x}{1+x}$, $(x^2+1)y' + y^2 + 1 = 0$; [6]

(b) $\phi(x) = ln(-x)$, $x < 0$, $xy' = 1$. [5]

A4. (a) Show that the equation $\dfrac{dy}{dx} = \dfrac{x^2}{1-y^2}$ is separable and then find an equation for the integral curves. [5]

(b) Prove that the equation $\dfrac{dy}{dx} = (x+y)^2$ can be reduced to an equation with separable variables type by uisng the substitution $v = x + y$. Obtain the solution if $y(0) = 0$. [6]

A5. Determine whether the equation $(3x^2 + 2y\sin 2x)dx + (2\sin^2 x + 3y^2)dy = 0$ is exact. Solve the equation. [5]

SECTION B: Answer THREE questions in this section [60].

B6. (a) Define a first order linear differential equation. [2]

(b) Solve $y' + \frac{1}{x}y = 2$, $x \neq 0$. [8]

(c) Find the Wronskian of the functions:

(i) $y_1(t) = sin(t)$ and $y_2(t) = 2sin(t)$, [5]

(ii) $y_1(t) = sin(t)$ and $y_2(t) = tsin(t)$. [5]

B7. (a) State the Lipschitz condition. [7]

(b) Show that the equation $\dfrac{2xy+1}{y}dx + \dfrac{y-x}{y^2}dy = 0$, $y \neq 0$ is exact. Hence find the solution. [8]

(c) Verify the existence and and uniqueness theorem for the differential equation $\dfrac{dy}{dx} = \dfrac{-x}{y}$, $y(0) = 1$. [5]

B8. Solve the following equations using the Method of Undetermined Co-efficients

(a) $2y'' + 2y' + 3y = x^2 + 2x - 1$, [10]

(b) $y'' + 3y' - 4y = \sin(2x)$. [10]

B9. Solve the following equations using the Method of Variation of Parameters

(a) $y'' + 4y = \sin^2(2x)$, [8]

(b) $x^5y'' + 6x^5y' + 9x^5y = x^{-5}e^{-3x}$. [12]

END OF QUESTION PAPER