NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA1211: MATHEMATICS FOR SCIENCE 11

MARCH 2025 EXAMINATION

Time : 3 hours

Answer **ALL** Questions in **SECTION A** [**40 MARKS**] and **ANY THREE** Questions from **SECTION B** [**60 MARKS**].

SECTION A

ANSWER ALL QUESTIONS [40 MARKS]

A1. (a) Solve the following ordinary diffferential equations

(i) $\dfrac{dy}{dx} = -2xy.$ [3]

(ii) $\dfrac{dy}{dt} = \dfrac{y}{t} + 1 + t; \qquad y(2) = -4\ .$ [4]

(b) Use the double integral to find the area of the region $\Re$ enclosed between the parabola $y = \frac{x^2}{2}$ and the line $y = 2x$. [4]

A2. Find and classify all the stationary points of $z = x^3 + y^3 - 3x - 12y + 20$. Evaluate z at each stationary point and state the maximum value of z. [6]

A3. Use Euler's method to find an approximate numerical value of y at $x = 0.3$ in steps of 0.1; given the IVP $y' = x + y, y(0) = 1$. [5]

A4. Solve the equation $4y'' + 4y' + y = 2\sin 3x$. [5]

A5. By reversing the order of integral evaluate $\int_0^1 \int_0^{\cos^{-1} y} x dx dy$. [5]

A6. When a switch is closed in a circuit containing a volatage source E, a resistance R and an inductance L, the current i builds up at a rate given by $L\frac{di}{dt} + Ri = E$. Find i as a function of t. How long will it be, before the current reaches half of its maximum value if $E = 6\ volts$, $R = 100\ ohms$ and $L = 0.1\ henry$? [5]

SECTION B

ANSWER ANY THREE QUESTIONS [60 MARKS]

B7. (a) Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dy dx$. [5]

(b) (i) Show that the differential equation of the type $\frac{dy}{dx} = f(\frac{y}{x})$ can be solved by the variables seperables technique if we make the substitution $\frac{y}{x} = v$ *i.e.* $y = vx$, where v is a differentiable function of x. [4]

(ii) Hence find the general solution of $xy' = x\sec(\frac{y}{x}) + y$. [5]

(c) Newton's law of cooling states that the rate at which a body cools is proportional to the excess of its temperarure above that of its surroundings. A body, initially at 80^0C, is cooling in surroundings maintained at 20^0C. After 10 minutes the temperature of the body is 60^0C. Find the body's temperature after further 10 minutes. [6]

B8. (a) Solve the IVP, $y' = \frac{y}{x} + 2x^2$, $\qquad y(1) = 2$. [4]

(b) Use the modified-Euler method to obtain the approximate solutions of the problem in (a) over the interval $[1; 1.4]$ with step size $h = 0.2$. [5]

(c) Use the Runge-Kutta method of order four (RK4) to obtain the approximate solutions of the problem in (a) over the interval $[1; 1.4]$ with step size $h = 0.2$. [8]

(d) Compare the results in (b) and (c) with the exact solution. [3]

[Hint: The formula for Runge-Kutta method of order four is given by
$y_{k+1} = y_k + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$,
where
$k_1 = f(x_k, y_k)$
$k_2 = f(x_k + \frac{1}{2}h, y_k + \frac{1}{2}hk_1)$
$k_3 = f(x_k + \frac{1}{2}h, y_k + \frac{1}{2}hk_2)$
$k_4 = f(x_k + h, y_k + hk_3)$.]

B9. (a) Find the general solution of the IVP's

(i) $y'' - 8y' + 16y = 6e^{4t}, \quad y(0) = 1, \quad y'(0) = 3.$ [6]

(ii) $y'' - 4y' + 8y = 13\sin t, \quad y(0) = 0, \quad y'(0) = 1.$ [6]

(b) Let $g(x, y, z) = xyz + \sin(x^2 + y)$ where $x = \cos t, \quad y = 1 + t^2$ and $z = lnt$. Use the Chain rule to find $\frac{\partial g}{\partial t}$. [4]

(c) Find the curl and divergence of $\vec{F}$ if $\vec{F} = xi + y^2\mathbf{j} + xz^3\mathbf{k}$. [4]

B10. (a) Verify that $y_1 = e^{-2x}$ and $y_2 = xe^{-2x}$ are solutions of the differential equation $y'' + 4y' + 4y = 0$. [4]

(b) Prove that $\int_0^1 \int_0^1 \frac{x - y}{(x + y)^3} dydx = \frac{1}{2}$. [5]

(c) Using double integrals, find the volume of the region $\Re$ common to the intersecting cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$. [5]

(d) Solve the equation $y'' + 9y = 18x + 27 + 2e^{3x}$. [6]

END OF QUESTION PAPER