NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA1211: MATHEMATICS FOR SCIENCE 11

AUGUST 2024 SPECIAL SUPPLEMENTARY EXAMINATION

Time : 3 hours

Answer **ALL** Questions in **SECTION A [40 MARKS]** and **ANY THREE** Questions from **SECTION B [60 MARKS]**.

SECTION A

ANSWER ALL QUESTIONS [40 MARKS]

A1. (a) Solve the following IVP's

(i) $\dfrac{dy}{dt} = \dfrac{\cos t}{\cos y}, \quad y(0) = \dfrac{\pi}{6}.$ [4]

(ii) $\dfrac{dy}{dt} = \dfrac{y}{t} + 1 + t; \quad y(2) = -4\ .$ [4]

A2. A simple closed electrical circuit contains an inductance L and a resistance R in series, and current I is caused to flow by the application of voltage V across two terminals located between resistance and inductance. The equation governing this current is in the following equation

$$\frac{dI}{dt} + \frac{R}{L}I = \frac{V}{L}, \quad I(0) = 0$$

(a) Find the current flowing, $I(t)$. [4]

(b) Find $\lim_{t \to \infty} I(t)$. [2]

(c) How long will it take after the switch is closed for the current to reach 90% of its limiting value. [3]

A3. Solve the equation $y'' + 6y' + 9y = 4e^{-3t}, \quad y(0) = 2, \quad y'(0) = -5.$ [5]

A4. Given the IVP $y' = x - y, y(0) = 1$, apply Euler's method to find the first five approximate numerical values of y on interval $[0, 1]$ in steps of 0.2. [5]

A5. Find and classify the stationary points of the function $f(x, y) = x^3 - y^3 - 3xy + 4$. [5]

A6. Evaluate $\int_0^1 \int_y^0 e^{x^2} dxdy.$ [4]

A7. Find the equation of the tangent plane and normal line to the graph $z = 4x + y^2$ at $(-1, 3, 5)$. [4]

SECTION B

ANSWER ANY THREE QUESTIONS [60 MARKS]

B8. (a) Consider the equation $y'' + y' - 12y = 0$. Verify that $y_1 = e^{3t}$ and $y_2 = e^{-4t}$ are solutions and hence find the solution satisfying $y(0) = 1, \quad y'(0) = 2.$ [4]

(b) Find the solution to $y'' - 4y' + 8y = 13 \sin t, \quad y(0) = 0, \quad y'(0) = 1.$ [6]

(c) Solve $(1 + x^2y^2)y + (yx - 1)^2y' = 0$, using the substitution $xy = t$. [5]

(d) Find the directional derivative of $f(x, y, z) = xy^2 - 4x^2y + z^2$ at $(1, -1, 2)$ in the direction of $6\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$. [5]

B9. (a)(i) Use the double integral to find the area of the region $\mathbb{R}$ enclosed between the parabola $y = \dfrac{x^2}{2}$ and the line $y = 2x$. [6]

(ii) If $f(x, y) = x^5 + y^5 - x^2 - y^2 + 1$, find all points at which f can have absolute minimum or maximum. Consider each point and classify it. [9]

(b) Evaluate $\int_0^4 \int_y^{2y} (8x + e^y)dxdy.$ [5]

B10.(a) Consider the initial value problem $y' = y^2 + 1, \quad y(0) = 0$. Use the modified Euler method with $h = 0.1$ to generate a solution of the initial value problem, correct to seven decimal places, when $x = 0.4$. [8]

(b) Solve the IVP, $y' = 1 - x + 4y, \quad y(0) = 1.$ [4]

(c) Use the Runge-Kutta method of order four (RK4) to obtain the approximate the value of y at $x = 0.4$, for the problem in (b) above, with step size of $h = 0.2$. [8]

[Hint: The formula for Runge-Kutta method of order four is given by
$y_{k+1} = y_k + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$,
where
$k_1 = f(x_k, y_k)$,
$k_2 = f(x_k + \frac{1}{2}h, y_k + \frac{1}{2}hk_1)$,
$k_3 = f(x_k + \frac{1}{2}h, y_k + \frac{1}{2}hk_2)$,
$k_4 = f(x_k + h, y_k + hk_3)$].

B11.(a) By reversing the order of integral evaluate $\int_0^1 \int_0^{\cos^{-1} y} x\,dx\,dy$. [5]

(b) Solve the equation $y'' + 9y = 18x + 27 + 2e^{3x}$. [5]

(c) Given that $\vec{F} = (x + 2y + az)\mathbf{i} + (bx - 3y - z)\mathbf{j} + (4x + cy + 2z)\mathbf{k}$

(i) Find the constants a, b, c so that $\vec{F}$ is irrotational. [2]

(ii) Show that $\vec{F}$ can be expressed as a gradient of a scalar function, that is, find a scalar ϕ such that $\vec{F} = \nabla\phi$. [3]

(d) Using double integrals, find the volume of the region $\mathbb{R}$ common to the intersecting cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$. [5]

END OF QUESTION PAPER