NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA1216

FACULTY OF APPLIED SCIENCE

DEPARTMENT OF APPLIED MATHEMATICS

SMA1216: ENGINEERING MATHEMATICS IB

SUPP EXAMINATION 2024

Time : 3 hours

Candidates should attempt all questions from Section A [**40 MARKS**] and ANY THREE questions in Section B [**60 MARKS**].

SECTION A:[40].

A1. Define the following:

(a) Directional derivative, [2]

(b) Diagonal matrix, [2]

(c) Ordinary differential equation. [2]

A2. Show that,

$$\lim_{(x,y)\to(0,0)} (x+1)^2 + 2(y+2) = 5.$$

[5]

A3. Solve the following system using Crame's method,

$$\begin{aligned} 3y + 2x &= z + 1, \\ 3x + 2z &= 8 - 5y, \\ 3z - 1 &= x - 2y. \end{aligned}$$

[5]

A4. Determine the set of points at which the function

$$f(x,y) = \begin{cases} \dfrac{x^2y^3}{2x^2+2y^2}, & (x,y) \neq (0,0) \\ \dfrac{1}{2}, & (x,y) = (0,0) \end{cases}$$

is continuous. [8]

A5. Let u, v and w be independent vectors. Show that $u+v, u-v$ and $u-2v+w$ are also independent. [5]

A6. Solve the following differential equations,

(a) $\dfrac{dy}{dt} = 1 + \dfrac{y}{t} + \dfrac{y^2}{t^2}$. [4]

(b) $(4x^3y - 15x^2 - y)dx + (x^4 + 3y^2 - x)dy = 0$. [7]

SECTION B: [60].

Answer any **THREE** questions from this section **[60 MARKS]**

B7. (a) Find the eigenvalues and the corresponding eigenvectors of the matrix

$$A = \begin{pmatrix} -1 & 7 & -1 \\ 0 & 1 & 0 \\ 0 & 15 & -2 \end{pmatrix}.$$

[10]

(b) Consider the following system,

$$\begin{aligned} x - 2y &= 1, \\ x - y + az &= 2, \\ ay + 9z &= b, \end{aligned}$$

(i) find the values of a and b for which the system has a unique solution, [5]

(ii) and find points (a,b) such that the system has more than one solution. [5]

B8. (a) Use Lagrange multipliers to find the extreme values of the function $f(x,y,z) = 4x^2y^2z^2$ inside the region $x^2+y^2+z^2=9$. [10]

(b) Use the chain rule to find $\dfrac{\partial^2 z}{\partial s^2}$ given that $z(x,y) = \sin(x^2y)$, where $x = st^2$, and $y = s^2 + \dfrac{1}{t}$. [10]

B9. (a) Use the ϵ - δ definition to prove that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{5x^2 - 5y^2}{x^4 - y^4} = 1.$ [6]

(b) Find the equation of the tangent plane and normal line at the point $(4,-1,1)$ to the surface $x^2 + 2y^2 + 3z^2 = 21$. [6]

(c) Use the concept of the differential to find an approximation of to

$$\sqrt[5]{\frac{33}{241}}.$$

[4]

(d) Suppose that over a certain region of space the electrical potential V is given by $V(x,y,z) = 5x^2 - 3xy + xyz$. Find the maximum rate of change of the potential at $P(3,4,5)$. [4]

B10. Solve the following differential equation,

(a) $(1+x^2)dy + (xy + x^3 + x)dx = 0$, [4]

(b) $y'' - 2y' - 3y = 4e^{2x} + 2x^3$, using undetermined coefficients method. [7]

(c) $y'' + 5y' - 6y = 10e^{2x}$, using variation of parameters. [9]

END OF QUESTION PAPER