

FACULTY OF APPLIED SCIENCE  
DEPARTMENT OF APPLIED MATHEMATICS  
SMA1216: ENGINEERING MATHEMATICS IB

MARCH INTAKE EXAMINATION 2024

Time : 3 hours

Candidates should attempt all questions from Section A [40 MARKS] and ANY THREE questions in Section B [60 MARKS].

SECTION A:[40].

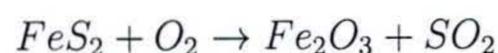
A1. Define the following:

- (a) Diagonal matrix, [2]  
(b) Orthogonal matrix. [2]

A2. Sketch the domain of the function:  $f(x, y) = \frac{xy}{\sqrt{\ln(x^2 + y^2 - 16)}}$  [6]

A3. (a) Compute  $\lim_{(x,y) \rightarrow (1,1)} \frac{y^2 + xy - 2x^2}{y - x}$ . [2]  
(b) Using the delta-epsilon definition verify your answer in A3(a). [6]

A4. Balance the following chemical equation used in the production of sulphuric acid using matrix method:



[5]

A5. Determine whether the function  $f(x, y) = \frac{x + y}{\sqrt{x^2 + y^2 - 25}}$  is continuous on the region  $(x - 2)^2 + y^2 \leq 1$ . [5]

A6. Let  $u, v$  and  $w$  be independent vectors. Show that  $u + v, u - v$  and  $u - 2v + w$  are also independent. [5]

A7. Solve the following differential equation:

$$(x^4 + 3y^2 - x)dy - (15x^2 - 4x^3y + y)dx = 0.$$

[7]

**SECTION B: [60].**

Answer any **THREE** questions from this section [60 MARKS]

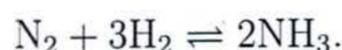
B8. (a) Find  $A^6$  using Diagonal matrix of  $A$ .

$$A = \begin{pmatrix} -1 & 7 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

[10]

(b) Abby, Ben, and Carter have a total of \$47 with them. Ben has half the amount of money that Abby has, and Carter has \$3 more than Ben. Using the LU decomposition find how much money do they each have. [10]

B9. (a) The Haber-Bosch process produces ammonia by a direct union of nitrogen and hydrogen under conditions of constant pressure  $P$  and constant temperature:



The partial pressures  $x, y,$  and  $z$  of hydrogen, nitrogen, and ammonia satisfy  $x + y + z = P$  and the equilibrium law  $\frac{z^2}{xy^3} = k$ , where  $k$  is a constant. The maximum amount of ammonia occurs when the maximum partial pressure of ammonia is obtained. Find the maximum value of  $z$ . [10]

(b) Use the chain rule to find  $\frac{\partial^2 z}{\partial s^2}$  given that  $z(x, y) = \sin(x^2y)$ , where  $x = st^2$ , and  $y = s^2 + \frac{1}{t}$ . [10]

B10. (a) Use the  $\epsilon$  -  $\delta$  definition to prove that  $\lim_{(x,y) \rightarrow (0,0)} \frac{5x^2 - 5y^2}{x^4 - y^4} = 1$ . [6]

(b) Find the equation of the tangent plane and normal line at the point  $(4, -1, 1)$  to the surface  $x^2 + 2y^2 + 3z^2 = 21$ . [6]

(c) Use the concept of the differential to find an approximation of to

$$\sqrt[5]{\frac{33}{241}}$$

[4]

(d) Suppose that over a certain region of space the electrical potential  $V$  is given by  $V(x, y, z) = 5x^2 - 3xy + xyz$ . Find the maximum rate of change of the potential at  $P(3, 4, 5)$ . [4]

B11. Solve the following differential equation,

(a)  $y'' + 5y' - 6y = 10e^{2x}$  for  $y(0) = 1$  and  $y'(0) = 2$ , using variation of parameters. [9]

(b)  $(x^2 + 2)y'' + 3xy' - y = 0$ , using power series solutions. [11]

END OF QUESTION PAPER