NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA1216

FACULTY OF APPLIED SCIENCE

DEPARTMENT OF APPLIED MATHEMATICS

SMA1216: ENGINEERING MATHEMATICS IB

MARCH INTAKE EXAMINATION 2024

Time : 3 hours

Candidates should attempt all questions from Section A [**40 MARKS**] and ANY THREE questions in Section B [**60 MARKS**].

SECTION A:[40].

A1. Define the following:

(a) Diagonal matrix, [2]

(b) Orthogonal matrix. [2]

A2. Sketch the domain of the function: $f(x,y) = \dfrac{xy}{\sqrt{In(x^2+y^2-16)}}$ [6]

A3. (a) Compute $\displaystyle\lim_{(x,y)\to(1,1)} \frac{y^2+xy-2x^2}{y-x}$. [2]

(b) Using the delta-epsilon definition verify your answer in **A3(a)**. [6]

A4. Balance the following chemical equation used in the production of sulphuric acid using matrix method:

$$FeS_2 + O_2 \to Fe_2O_3 + SO_2$$

[5]

A5. Determine whether the function $f(x,y) = \dfrac{x+y}{\sqrt{x^2+y^2-25}}$ is continuous on the region $(x-2)^2+y^2 \leq 1$. [5]

A6. Let u, v and w be independent vectors. Show that $u+v, u-v$ and $u-2v+w$ are also independent. [5]

A7. Solve the following differential equation:

$$(x^4+3y^2-x)dy-(15x^2-4x^3y+y)dx=0.$$

[7]

SECTION B: [60].

Answer any **THREE** questions from this section **[60 MARKS]**

B8. (a) Find A^6 using Diagonal matrix of A.

$$A = \begin{pmatrix} -1 & 7 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

[10]

(b) Abby, Ben, and Carter have a total of \$47 with them. Ben has half the amount of money that Abby has, and Carter has \$3 more than Ben. Using the **LU** decomposition find how much money do they each have. [10]

B9. (a) The Haber–Bosch process produces ammonia by a direct union of nitrogen and hydrogen under conditions of constant pressure P and constant temperature:

$$N_2 + 3H_2 \rightleftharpoons 2NH_3.$$

The partial pressures x, y, and z of hydrogen, nitrogen, and ammonia satisfy $x+y+z=P$ and the equilibrium law $\frac{z^2}{xy^3}=k$, where k is a constant. The maximum amount of ammonia occurs when the maximum partial pressure of ammonia is obtained.Find the maximum value of z. [10]

(b) Use the chain rule to find $\dfrac{\partial^2 z}{\partial s^2}$ given that $z(x,y)=\sin(x^2y)$, where $x=st^2$, and $y=s^2+\dfrac{1}{t}$. [10]

B10. (a) Use the ϵ - δ definition to prove that $\lim_{(x,y)\to(0,0)} \frac{5x^2 - 5y^2}{x^4 - y^4} = 1.$ [6]

(b) Find the equation of the tangent plane and normal line at the point $(4, -1, 1)$ to the surface $x^2 + 2y^2 + 3z^2 = 21$. [6]

(c) Use the concept of the differential to find an approximation of to

$$\sqrt[5]{\frac{33}{241}}.$$

[4]

(d) Suppose that over a certain region of space the electrical potential V is given by $V(x, y, z) = 5x^2 - 3xy + xyz$. Find the maximum rate of change of the potential at $P(3, 4, 5)$. [4]

B11. Solve the following differential equation,

(a) $y'' + 5y' - 6y = 10e^{2x}$ for $y(0) = 1$ and $y'(0) = 2$, sing variation of parameters. [9]

(b) $(x^2 + 2)y'' + 3xy' - y = 0$, using power series solutions. [11]

END OF QUESTION PAPER