NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA1216

FACULTY OF APPLIED SCIENCE

DEPARTMENT OF APPLIED MATHEMATICS

SMA1216: ENGINEERING MATHEMATICS IB

MARCH 2025 EXAMINATION

Time : 3 hours

Candidates should attempt all questions from Section A [**40 MARKS**] and ANY THREE questions in Section B [**60 MARKS**].

SECTION A:[40].

A1. Define the following:

(a) Diagonal matrix. [2]

(b) Upper triangular matrix. [2]

(c) Domain and range of a function $f(x, y)$. [3]

A2. Solve the following system of equations,

$$\begin{aligned} x_1 - 2x_2 + x_3 - 4x_4 &= 1, \\ x_1 + 3x_2 + 7x_3 + 2x_4 &= 2, \\ x_1 - 12x_2 - 11x_3 - 16x_4 &= 5. \end{aligned}$$

[7]

A3. Find the relative extrema given

$$f(x, y) = 2x^2 + 4y^2 - 2xy - 10x - 2y + 2.$$

[7]

A4. Verify whether the following functions $f_1(x) = e^{-2x}$, $f_2(x) = e^{-x}$ and $f_3(x) = e^{-2x} sin(e^x)$ qualify to be solutions of a given differential equation. [6]

A5. Solve the following differential equations:

(a) $x\frac{dy}{dx} + 2y = 4x^2$. [6]

(b) $(x^2 - x)\, y'' + (x - 2)y' = 0, y(1) = 2$. [7]

SECTION B: [60].

Answer any **THREE** questions from this section [**60 MARKS**]

B6. (a) Determine the eigenvalues and the corresponding eigenvectors of the following matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 8 & 1 & 3 \end{pmatrix}.$$

[10]

(b) Consider the following system of equations,

$$\begin{aligned} x + 2y - z &= 1, \\ 2x + 3y + z &= 1, \\ -4x - 5y + (k^2 - 9)z &= k + 1. \end{aligned}$$

Find the values of k for which the system has

(i) a unique solution. [3]

(ii) no solution. [3]

(iii) infinitely many solutions. [4]

B7. (a) Use the $\epsilon - \delta$ definition to prove that $\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2+y^2} = 0$. [6]

(b) Determine the set of points at which the function

$$f(x, y) = \begin{cases} \frac{x^2-3y^2}{x^2+2y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

is continuous. [8]

(c) Find the equation of the tangent plane and normal line at the point $(3, 4, 5)$ to the surface $x^2 + y^2 - z^2 = 0$. [6]

B8. (a) Use Lagrange multipliers to find the extreme values of the function $f(x, y, z) = 4x^2y^2z^2$ subject to $x^2 + y^2 + z^2 = 9$. [10]

(b) Use the chain rule to find $\frac{\partial w}{\partial t}$ and $\frac{\partial w}{\partial \theta}$ given that $w = (u^2 + v^2)^{\frac{3}{2}}$, $u = e^{-t} \sin\theta$ and $v = e^{-t} \cos\theta$. [6]

(c) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if z is defined implicitly as a function of x and y by the equation $x^3 + y^3 + z^3 + 6xyz = 1$. [4]

B9. Solve the following differential equations,

(a) $y' = \frac{xy^3}{\sqrt{1+x^2}}$ for $y(0) = -1$ and find the interval of validity of the solution. [5]

(b) $y'' + 4y' + 4y = -10xe^x + 5sin(x)$, using the method of undetermined coefficients. [7]

(c) $4y'' - 4y' + y = e^{\frac{x}{2}}\sqrt{1 - x^2}$, using variation of parameters. [8]

END OF QUESTION PAPER