NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA2103

FACULTY OF APPLIED SCIENCE

DEPARTMENT OF APPLIED MATHEMATICS

SMA2103: THEORETICAL MECHANICS

DECEMBER 2024 EXAMINATION

Time : 3 hours

Candidates should attempt **ALL** questions from Section A[**40 MARKS**] and **ANY THREE** questions from Section B[**60 MARKS**].

SECTION A (40 MARKS)

A1. A man makes a rectilinear motion away from the foot of an upright and isolated street light.

(a) Develop a mathematical model for the velocity of the tip of the man's shadow v and make an analysis on its magnitude relative to his own velocity u clearly defining all your parameters. [5]

(b) State the main assumptions of your model. [1]

(c) If the shadow tip velocity is 1.25 times faster than the man's velocity, find the height of the street light given that the man is 1.8m tall. [2]

A2. Define the following and give an appropriate example in each case.

(a) Non-inertial frame of reference. [3]

(b) Holonomic constraint. [3]

(c) Field of force. [3]

A3. Determine whether the force field $\mathbf{F} = -kr^3\mathbf{r}$ is conservative or not. [4]

A4. A particle of mass 5 moves along, $\mathbf{r} = (2t^3 + t)\mathbf{i} + (3t^4 - t^2 + 8)\mathbf{j} - 12t^2\mathbf{k}$.
Find the

(a) power applied to the particle by the force field. [3]

(b) angular momentum ($\mathbf{L}$) about the origin and prove that the torque $\tau = \dot{\mathbf{L}}$. [7]

A5. (a) Find $|\overline{\mathbf{r}}|$ for a system of 3 discrete particles of masses $3, 5$ and 2 located at $(1; 0; -1), (-2; 1; 3)$ and $(3; -1; 1)$ respectively. [3]

(b) Show that if I_{Cz} is the moment of inertia of a rigid body about the Cz-axis which passes through the center of mass and parallel to $C'z'$, then

$$I_{C'z'} = I_{Cz} + Md^2.$$

[6]

SECTION B

B6. (a) Define 'relative motion' of a particle M which is moving in a moving reference system. [2]

(b) The absolute acceleration $\mathbf{a}_{abs}$ of a particle in a rotational frame is defined as the sum of $\dot{\mathbf{v}}_{tr}$ and $\dot{\mathbf{v}}_{rel}$ as given below.

$$\mathbf{a}_{abs} = \frac{d\mathbf{v}_{tr}}{dt} + \frac{d\mathbf{v}_{rel}}{dt},$$

where,

$$\frac{d\mathbf{v}_{tr}}{dt} = \dot{\mathbf{v}}_{tr} = \mathbf{a}_{tr} + (\omega \times \mathbf{v}_{rel}).$$

If $\mathbf{v}_{rel} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j} + \dot{z}\mathbf{k}$, where $\dot{x}$, $\dot{y}$ and $\dot{z}$ are time derivatives.
Find $\dot{\mathbf{v}}_{rel}$ and show that, $\mathbf{a}_{abs} = \mathbf{a}_{tr} + \mathbf{a}_{rel} + \mathbf{a}_{cor}$, where $\mathbf{a}_{cor} = 2(\omega \times \mathbf{v}_{rel})$ is the Coriolis acceleration. [5]

(c) Show that if a body is released from a *small* height H above the surface of the earth to fall freely then it is deflected from the vertical due to the earth's rotation by a magnitude of

$$x = \frac{2}{3}\omega \cos\lambda \sqrt{\frac{2H^3}{g}},$$

where ω is the earth's angular velocity, λ is the latitude, H is the height of the fall and g is acceleration due to gravity. [13]

B7. (a) State the Newton's laws of motion. [1]

(b) The acceleration of a particle moving along a space curve can be expressed as

$$\mathbf{a} = \kappa v^2 \mathbf{N} + \frac{dv}{dt}\mathbf{T},$$

where κ is the curvature and v is the speed. Consider a space curve C defined by, $\mathbf{r} = t(\mathbf{i} + \frac{t}{2}\mathbf{j} + \mathbf{k})$. To this space curve, find the:

(i) Force experienced by a particle of mass 2. Giving your answer in the form $\mathbf{F} = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$. [15]

(ii) Bi-normal vector. [4]

B8. (a) Use definitions to distinguish between a particle and a body. [2]

(b) The velocity $\dot{y}$ of a particle falling in a resisting medium proportional to the first power of velocity is given by $\dot{y} = \frac{g}{k}(1 - e^{-kt})$.
Find the maximum velocity. [2]

(c) Show that the equation of motion of a particle mass m falling through a resisting medium proportional to the second power of velocity is given by,

$$y = \frac{1}{k}\ln\cosh\left((\sqrt{gk})t\right).$$

[13]

(d) Taking $g = 9.8052 m/s^2$ and $k = 0.2$ find the time t it takes the particle to hit the ground if it is released from a height of 50 meters. [3]

B9. (a) If the Lagrangian function $L = T - V$ where T and V are defined as usual, then show that for a conservative system we have,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_\alpha}\right) - \frac{\partial L}{\partial q_\alpha} = 0.$$

[5]

(b) Set up the Lagrangian for a simple pendulum and obtain an equation describing its motion. [8]

(c) (i) Define a central force. [2]

(ii) Set up the Hamiltonian for a particle moving under the influence of a central force field in the xy plane. [5]

END OF QUESTION PAPER