

NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY  
SMA2109

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA2109: RESEARCH METHODS IN MATHEMATICS

DECEMBER 2024 EXAMINATION

Time : 3 hours

Candidates should attempt **ALL** questions from Section A[25 MARKS] and **ANY THREE** questions from Section B[75 MARKS].

**SECTION A**

**A1.** Define the following terms

- (a) Research process. [2]
- (b) Quantitative data. [2]

**A2.** Name and describe four most common methods of survey distribution. [4]

- A3.** (a) Outline the practical steps in a research process. [5]  
(b) Explain what is qualitative data and state the disadvantages of qualitative data. [4]

- A4.** If the growth rate of the number of bacteria at any time  $t$  is proportional to the number present at  $t$  and doubles in 1week,
- (a) how many bacteria can be expected after 2 weeks?. [4]
  - (b) how many bacteria can be expected after 4 weeks?. [4]

## SECTION B

- B5.** (a) Four managers were assigned to do 4 tasks. The number of hours taken by each manager to finish each task are presented in the following table.

Managers	1	2	3	4
M1	10	6	12	8
M2	15	18	5	11
M3	17	10	13	16
M4	14	12	13	10

- (i) Formulate a model to minimize the total time taken to finish all the four tasks by the four managers. [5]
- (ii) Solve the above problem using the Hungarian method, and help the management in allocating the four tasks to the four managers and minimize the time taken to finish the tasks. [7]
- (b) A radioactive substance decays at a rate proportional to its mass. When the mass of a sample is  $0.030g$ , the decay rate is  $0.002g$  per day.
- (i) Write down a differential equation and solve it to give mass in terms of time for these initial conditions. [6]
- (ii) Find the time the sample takes to decay to  $0.010g$ . [2]
- (iii) Find the time the sample takes to decay from  $0.012g$  to  $0.005g$ . [3]
- (iv) Find the rate of decay after 12 days. [2]
- B6.** Bread from an oven is placed in a cardboard box of initial temperature of  $20^{\circ}C$ . The rate at which the bread cools down is directly proportional to the difference between its temperature and that of the cardboard box.
- (a) Given that  $\theta^{\circ}C$  is the temperature of the bread at time  $t$  minutes, write down a differential equation, relating  $\theta$  and  $t$ . [4]
- (b) Find the general solution of the differential equation, expressing  $\theta$  in terms of  $t$ . [8]
- (c) Initially  $\theta = 100$  and  $\theta = 80$  when  $t = 10$ ,
- (i) find the particular solution of the differential equation. [4]  
Hence find
- (ii) the temperature of the bread at  $t = 20$ . [4]
- (iii) the time, to the nearest minute, at which the temperature of the bread has dropped by  $40^{\circ}C$ . [3]
- (iv) Hence, draw the graph of  $\theta$  against  $t$ . [2]

- B7.** Astra Paints Pvt Ltd produces both interior and exterior paints from two raw materials  $M1$  and  $M2$ . The following table provides basic data of the problem, including profit per ton of each type of paint produced.

	Tons of Raw Materials per Ton of		Maximum Available
	Interior paint	Exterior paint	
Raw material1(M1)	4	6	24
Raw material2(M2)	2	1	6
profit per ton(\$1000)	4	5	

A market research survey restricts the maximum daily demand of interior paint to 2 tons. Additionally the daily demand for interior paint cannot exceed that of exterior paint by more than 1 ton. Astra wants to determine the optimum product mix of interior and exterior paints that maximizes the total daily profit.

- (a) Formulate the linear programming model to maximize profit of the total paint produced daily. [5]
- (b) Find the optimal mix solution which maximizes profits for Astra Paints , using:
- (i) Graphical method. [5]
- (ii) Simplex method. [10]
- (c) Compare the two methods above and state the best method for the optimal solution. [5]
- B8.** (a) Suppose we have 3 plants (origins) producing a certain product with production capacities given in the table below,

Plants	Production capacity
P1	50
P2	40
P3	30
Total	120

Demand for the product at 3 retail outlets (destination) is as below;

Plants	Production capacity
R1	45
R2	15
R3	30
Total	90

The cost(\$) of transporting each unit product is presented in the table below.

Plants	R1	R2	R3
P1	10	6	7
P2	6	11	6
P3	12	7	11

- (i) Formulate the model to minimize the total cost of transporting all products from from the three plants to their destinations. [5]
- (ii) Solve using North-West Corner method. [5]
- (iii) Solve using the Least-Cost method. [5]
- (iv) Compare the two methods in (i) and (ii), and state the most accurate method to minimize transportation costs. [2]
- (b) Peter was arrested while leaving a bar and he claimed that he had been inside the bar for at least 30 minutes (which would provide him with an alibi). The police checked the water temperature of his car (parked near the entrance of the bar) at the instant of arrest and 30 minutes later, obtaining the values  $190^{\circ}F$  and  $110^{\circ}F$ , respectively. Do these results give Peter an alibi? (Solve by inspection). [4]
- (c) Name and explain any three principles of administering a survey to collect quantitative data. [4]