NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCE

DEPARTMENT OF APPLIED MATHEMATICS

SMA2116: ENGINEERING MATHEMATICS II

August 2024 Special Examination

Time : 3 hours

Candidates should attempt **ALL** questions from **Section A** (40 marks) and **ANY THREE** questions from **Section B** (20 marks each).

SECTION A [40]

A1. Evaluate the following by first changing the order of integration $\int_0^1 \int_{2y}^2 e^{-\frac{y}{x}}\, dxdy$. [5]

A2. (a) Define a solenoidal vector. [2]

(b) Show that the vector function $\mathbf{F} = (6xy^2 + z)\mathbf{i} + (6x^2y + 3y^2)\mathbf{j} + x\mathbf{k}$ is conservative and find its potential. [5]

A3. Verify the Green's theorem for $\oint_c (xy + y^2)dx + x^2dy$, where c is the region bounded by $y = x$ and $y = x^2$. [6]

A4. (a) Find the directional derivative of $f(x, y) = \dfrac{xy}{x+y}$ at a point $(2, -3)$ in the direction $3\mathbf{i} - 8\mathbf{j}$. [4]

(b) Show that $\int\int\int_{R(x,y,z)} dxdydz = \int\int\int_{R(\rho,\phi,\theta)} \rho^2 sin\phi d\rho d\phi d\theta$ by change of variable from (x, y, z) coordinate system to the (ρ, ϕ, θ) coordinate system according to the transformation $x = \rho sin\phi cos\theta$, $y = \rho sin\phi sin\theta$ and $z = \rho cos\phi$. [8]

A5. Find the Fourier Series for $f(x) = \pi - x$ for $-\pi < x < \pi, f(x + 2\pi) = f(x)$. [10]

SECTION B [60]

B6. (a) Using polar coordinates, evaluate $\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dydx$. [4]

(b) If $\mathbf{F}$ is irrotational, prove that $\mathbf{F}$ is conservative. [6]

(c) Verify the divergence theorem for $\mathbf{F} = 4x^2\mathbf{i} - 2y^2\mathbf{j} + z\mathbf{k}$ taken over the region bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 2$. [10]

B7. (a) Verify that $\mathbf{F} = (3x^2 - 3yz + 2xz)\mathbf{i} + (3y^2 - 3xz + z^2)\mathbf{j} + (3z^2 - 3xy + x^2 + 2yz)\mathbf{z}$, represents a conservative field of force. [4]

(b) By changing to cylindrical coordinates, evaluate

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{0}^{\sqrt{4-x^2-y^2}} dzdydx.$$

[6]

(c) Verify Stoke's theorem for $\mathbf{F} = (2x - y)\mathbf{i} - yz^2\mathbf{j} - y^2z\mathbf{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. [10]

B8. (a) Evaluate $\int_c xyz^2 ds$, where c is the segment from point $(1, 1, 0)$ to $(2, 3, 1)$. [6]

(b) Expand $f(x) = 2x$, $o < x < 1$ in a half-range Fourier cosine series. [6]

(c) Define $r = \sqrt{x^2 + y^2 + z^2}$, show that $\nabla^2(\frac{1}{r}) = 0$. [8]

B9. (a) Find the volume of the solid S lying under the graph of the surface $z = x^3 + 4y$ and above the region R in the $xy-$plane bounded by the line $y = 2x$ and the parabola $y = x^2$. [6]

(b) Find the area bounded by $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$, given that the parametric equations are $x = acos^3\theta$ and $y = asin^3\theta$. [6]

(c) Evaluate $\int\int_{R_{xy}} (x + y)^2 dA$, where R_{xy} is a parallelogram with vertices at $A(0, 2)$, $B(-2, 0)$, $C(-6, 2)$ and $D(-4, 4)$. [8]

END OF QUESTION PAPER