NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA2116

FACULTY OF APPLIED SCIENCE

DEPARTMENT OF APPLIED MATHEMATICS

SMA2116: ENGINEERING MATHEMATICS II

December 2024 Examination

Time : 3 hours

Candidates should attempt **ALL** questions from **Section A** (40 marks) and **ANY THREE** questions from **Section B** (20 marks each).

SECTION A [40]

A1. Evaluate $\int_{-\infty}^{1}\int_{0}^{e^x} \frac{1}{1-lny}\, dydx$ by changing the order of integration. [5]

A2. (a) Determine the constant c so that the vector $\mathbf{V} = (x+3y)\mathbf{i} + (y-2z)\mathbf{j} + (x+cz)\mathbf{k}$ is solenoidal. [3]

(b) For this value, find the curl of V. [4]

A3. Find the work done by the force field $\mathbf{F}(x,y) = xe^y\mathbf{i} + y\mathbf{j}$ in moving a particle along the parabola $y = x^2$ from $(-1,1)$ to $(2,4)$. [6]

A4. Verify the Green's theorem for $\oint_c (xy+y^2)dx + x^2dy$, where c is the region bounded by $y=\sqrt{x}$ and $y=x^2$. [6]

A5. Convert the following integral into cylindrical coordinates,

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{x^2+y^2}^{\sqrt{x^2+y^2}} xyz\ dzdxdy.$$

[5]

A6. (a) Given that $f(x) = 4x$ over the interval $0 < x < \pi$.
Sketch $f(x)$ for $-3\pi < x < 3\pi$. [2]

(b) Find the Fourier half-cosine range for $f(x)$. [5]

(c) Hence find the expression for approximating π^2. [4]

SECTION B [60]

B7. (a) A vector A is called irrotational if curlA=0. Given
$A = (x + 2y + az)\mathbf{i} + (bx - 3y - z)\mathbf{j} + (4x + cy + 2z)\mathbf{k}$,

(i) Find the constants a, b, c so that A is irrotational. [4]

(ii) Find the scalar potential ϕ for A so that $A = \nabla\phi$. [6]

(b) Verify the divergence theorem for $F = 4x\mathbf{i} - 2y^2\mathbf{j} + z^2\mathbf{k}$ taken over the region bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. [10]

B8. (a) Define a curl of a vector. [2]

(b) Let a be a constant vector and $r = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Prove that
$\nabla \times [(r.r)a] = 2(r \times a)$. [4]

(c) Find the area bounded by $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$, given that the parametric equations are $x = acos^3\theta$ and $y = asin^3\theta$. [6]

(d) Evaluate $\int\int_{R_{xy}} (x + y)^2 dA$, where R_{xy} is a parallelogram with vertices at $A(0, 2)$, $B(-2, 0)$, $C(-6, 2)$ and $D(-4, 4)$. [8]

B9. (a) Evaluate $\int_C 4x^3 ds$ where c is the line segment from $(-2, -1)$ to $(1, 2)$. [5]

(b) Find the Fourier series representation of $f(x) = (cosx + sinx)^2$,
$-\pi \leq x \leq \pi$, $f(x + 2\pi) = f(x)$. [5]

(c) Verify Stoke's theorem for $\oint_c xydx + 2yzdy + xzdz$ where c is given by
$z = 1 - y$, $0 \leq y \leq 1$, $0 \leq x \leq 2$. [10]

B10. (a) Evaluate by changing the order of integration $\int_0^1 \int_x^1 x^2\sqrt{1+y^4}dydx$. [6]

(b) Given a contour C, prove that the area A is given by $A = \frac{1}{2}\oint_c xdy - ydx$. [4]

(c) Find the volume of the solid that is bounded by the graphs $z^2 = 3x^2 + 3y^2, x = 0, y = 0, z = 2$, in the first octant. [6]

(d) The heat in a rectangular brooder house is described by the equation $T(x, y) = 5 + 2x^2 + y^2$. Determine the direction that must be taken by a chick standing at a coordinate $(4, 2)$ so that it cools as rapidly as possible. [4]

END OF QUESTION PAPER