NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA2201

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA2201:COMPLEX ANALYSIS

March 2025: Examination

Time : 3 hours

Candidates should attempt **ALL** questions from **Section A** (40 marks) and **ANY THREE** questions from **Section** B (20 marks each). All The Best!

SECTION A [40]

A1. (a) Write the Cauchy-Riemann equations in polar coordinates. [2]

(b) Write $z = 2 + \pi i$ as a complex exponential. [3]

A2. Determine u and v if $\omega = u + iv$ and

(a) $\omega = zsinh(z)$, [4]

(b) $\omega = z^3 e^z$. [4]

A3. (a) Find the real constants p and q for which the function $f(z) = u + iv$ is analytic, where $u(x, y) = px^4 - 6x^2y^2 + y^4 - 5x$ and $v(x, y) = 4x^3y - 4xy^3 + qy$. [4]

(b) for these value of p and q, find an explicit formula for the differentiable function $f(z)$ in terms of z. [4]

A4. Find the Taylor's series about $z = 0$ for $f(z) = \sin z$ in terms of z. [6]

A5. Evaluate

(a) $I = \oint_{|z|=3} \frac{Sinh3z}{(z^2+1)^2} dz$, using the extended Cauchy's integral formula. [7]

(b) $I = \int_0^{\pi} \frac{1}{(3+\cos\theta)} d\theta$. [6]

SECTION B [60]

B6. (a) Evaluate the following:

(i)

$$I = \oint_{|z|=1} \frac{e^z}{z^2+2z} \mathrm{dz}.$$

[3]

(ii)

$$I = \oint_{|z|=2} \frac{\sin iz}{z^2-4z+3} \mathrm{dz}.$$

[5]

(b) Derive the extended Cauchy's integral formula. [7]

(c) Use the extended Cauchy's integral to evaluate

$$I = \oint_{|z|=2} \frac{\cosh z}{(z+1)^3(z-1)} \mathrm{dz}.$$

[5]

B7. (a) Derive the formula for the Taylor's series. [10]

(b) Show that

$$\int_{-\infty}^{+\infty} \frac{dx}{(x^2+1)^4} = \frac{5\pi}{16}.$$

[5]

(c) Evaluate

$$\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta$$

[5]

B8. (a) Use the ML theorem to bound the following function:

$I = \oint_c \frac{dz}{z(z-2)^3}$, where c is a circle, centre origin and radius 5 [3]

(b) Prove the Cauchy-Goursat theorem, $\oint_c f(z)dz = 0$ [5]

(c) Define the following terms:

(i) Pole [2]

(ii) Removable singularity [2]

(iii) Essential singularity [2]

(d) Show that for $b > 0$

$$\int_0^\infty \frac{\cos x}{x^2 + b^2} dx = \frac{\pi e^{-b}}{2b}$$

[6]

B9. (a) Given the Cauchy-Riemann equations in cartesian coordinates as $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$. Show that, in polar coordinates, the Cauchy-Riemann equations are given by $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$, where $x = r cos\theta$ and $y = r sin\theta$. [10]

(b) (i) Find the image of the line $y = 2x$ under the transformation $\omega = \frac{z+i}{z-i}$. [7]

(ii) Hence comment about the image of the point (1,2). [3]

END OF QUESTION PAPER