NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA2206

FACULTY OF APPLIED SCIENCE

DEPARTMENT OF APPLIED MATHEMATICS

SMA2206: NUMERICAL ANALYSIS

MARCH 2025: EXAMINATION

Time : 3 hours

Candidates should attempt **ALL** questions from **Section A** (40 marks) and **ANY THREE** questions from **Section B** (20 marks each).

SECTION A

Answer ALL questions [**40 MARKS**]

A1. (a) Describe the types of errors encountered in numerical analysis. [2]

(b) (i) Show that the fixed-point iteration has a first order convergence rate. [4]

(ii) Plot the graphs of $y = \cos x$ and $y = 3x - 1$. [3]

(iii) Use the fixed-point iteration to solve the nonlinear equations $\cos x - 3x + 1 = 0$, in the interval $[0, \pi/2]$ to within 10^{-5}. [5]

A2. Find the first four series of the Picard's iterative method to solve the initial value problem $y' = x + y$ given that $y(0) = 1$. [8]

A3. (a) Describe how the Gauss-Seidel method works. [4]

(b) Perform four iterations of the Gauss-Seidel method to the following system

$45x_1 + 2x_2 + 2x_3 = 58,$

$-3x_1 + 22x_2 + 2x_3 = 47,$

$5x_1 + x_2 + 20x_3 = 67.$ [6]

A4. Find the value of y at $x = 7.5$ for the given set of points $(5, 12), (6, 13), (7, 14), (8, 15)$. [8]

SECTION B

Answer ANY THREE questions [60 MARKS]

B5. (a) Show that the Newton-Raphson method has a second order convergence rate. [4]

(b) A drug administered to a patient produces a concentration in the blood stream given by $c(t) = Ate^{-\frac{t}{3}}$ milligrams per milliliter t hours after A units have been injected. The maximum safe concentration is $1mg/ml$.

(i) What amount should be injected to reach this maximum safe concentration and when does this maximum occur? [4]

(ii) An additional amount of this drug is to be administered to the patient after the concentration falls to $0.25mg/ml$. Determine, to the nearest minute, when this second injection should be given. [10]

(iii) State the advantages and disadvantages of the Newton-Raphson method. [2]

B6. (a) Derive the three-point central differentiation formula for evaluating $f'(x)$ using Lagrange polynomials. [10]

(b) Use the three-point formula you obtained in (a) above to estimate $f'(1.20)$ for the following data

x	1.05	1.10	1.15	1.20	1.25	1.30
$f(x)$	-1.709847	-1.373823	-1.119214	-0.9160143	-0.7470223	-0.6015966

[4]

(c) The five-point formula is given by

$$f'(x_0) = \frac{1}{12h}\left[f(x_0 - 2h) - 8f(x_0 - h) + 8f(x_0 + h) - f(x_0 + 2h)\right].$$

Estimate $f'(1.20)$ using the five-point formula. [4]

(d) The data given in (b) is generated using the function $f(x) = \tan 2x$. Compare and comment on the accuracy of the two methods. [2]

B7. (a) For a fixed $a \in \mathbb{R}$, let $\beta = \int_1^3 (9x^2 - 5ax^4)dx$. Use the Trapezoidal rule with $h = 0.5$ to estimate the values of a and β. [10]

(b) (i) Use the Gaussian quadrature with $n = 2$ and $n = 4$ to evaluate the integral $\int_1^2 x \ln x dx$. [8]

n	points (x_i)	weights (w_i)
1	0	2
2	± 0.577350	1
3	0	$\frac{8}{9}$
	± 0.774597	$\frac{5}{9}$
4	± 0.339981	0.652145
	± 0.861136	0.347855

(ii) The exact value for the integral is $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2$. Comment on the effect of n. [2]

B8. (a) (i) Use the modified Euler's method with $h = 0.1$ to estimate $y(0.3)$ for the initial-value problem
$y' = y + t$, $0 < t < 1$, $y(0) = 1$. [3]

(ii) Solve the problem in (i) above using fourth-order Runge-Kutta method. [6]

(iii) The exact solution is given by $y(t) = 2e^t - t - 1$. Compare and comment on the two schemes you used in (i) and (ii) above. [1]

(b) (i) Determine constants a, b, c, and d that will produce a quadrature formula
$$\int_{-1}^{1} f(x)dx = af(-1) + bf(1) + cf'(-1) + df'(1)$$
that has degree of precision 3. [10]

END OF QUESTION PAPER