NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA2217

FACULTY OF APPLIED SCIENCE

DEPARTMENT OF APPLIED MATHEMATICS

SMA2217: Engineering Mathematics III

August 2024: Special Examination

Time : 3 hours

Candidates should attempt **ALL** questions from Section A [**40 Marks**] and **ANY THREE** questions from Section B [**60 Marks**]. Additional material: **Laplace Tables and Statistical Tables**

SECTION A (40 marks)

A1. (a) Define the Laplace transform of a Heaviside function. [2]

(b) Derive the Laplace transform of $Cosh(at)$. [4]

(c) Find the inverse Laplace transform $f(t)$ for $F(s) = \dfrac{S+2}{S^2+8S+25}$. [5]

A2. (a) Explain the difference between qualitative and quantitative random variables and give two examples for each. [4]

(b) A group of 600 engineering students went to the shop at the student service centre after being given their payouts. 200 of these bought books, 140 bought clothes, 150 bought electrical gadgets and 110 bought other items. Construct a pie chart for the data. [4]

A3. (a) A coin is biased so that a head is twice as likely to occur as a tail. If the coin is tossed three times, what is the probability of getting exactly two heads? [6]

(b) The probability of a telesales representative making a sale on a customer is 0.15. Calculate the least number of calls that need to be made by the representative for the probability of at least 1 sale to exceed 0.95. [4]

A4. (a) A census done by the Ministry of Tertiary Education in a certain country revealed that on average there are 55 students per lecturer. However, a study carried out by the lectures trade union in 26 colleges shows a mean of 58 students per lecturer, with a standard deviation of 4. Test at the 0.05 level of significance whether the number of students per lecturer has increased. [6]

(b) A soft drink machine is regulated so that the amount of drink dispensed is approximately normally distributed with a standard deviation equal to 30ml. Find a 95% confidence interval for the mean of all drinks dispensed by the machine if a random sample of 36 drinks had an average content of 260ml. [5]

SECTION B

B5. A landlord is interested in seeing if his apartment rents are typical. Thus, he has taken a random sample of 11 apartment rental and apartment sizes of similar apartment complexes. The following data was obtained.

No. of bedrooms	2	1	3	2	2	2	2	1	1	2	2
Rent $(in millions)	2.30	1.95	4.50	3.10	2.18	1.85	3.40	2.45	1.25	3.50	2.86

(a) Find the least squares regression line $y = a + bx$. [10]

(b) Interpret the meaning of the values of a and b calculated in part (a). [2]

(c) Predict the rentals for a tenant using a 5 bedroomed apartment. [3]

(d) Calculate the coefficient of determination, r^2 and interpret it. [5]

B6. (a) Find the Laplace transform of $e^{-t}\cos^2(3t)$. [7]

(b) Solve the following differential equation, subject to the specified initial conditions

$$y'' + 6y' + 9y = 0 \quad y(0) = -3, \quad y'(0) = 10.$$ [6]

(c) Find the inverse Laplace transform $f(t)$, given $F(s) = \dfrac{8 - 6S}{16s^2 + 9}$. [7]

B7. According to some standard, the mean child support to be paid to custodial mothers should be \$740000 per month. A random sample of 340 custodial mothers taken recently showed that the mean child support paid to them is \$864000 with the standard deviation of \$140000.

(a) Test at $\alpha = 0.05$ whether the amount is different from \$740000. [10]

(b) Calculate a 95% confidence interval of the mean child support paid to custodial mothers. [8]

(c) State any two assumptions of the Binomial distribution. [2]

B8. (a) Using the axioms of probability:

(i) Prove that if $A = \emptyset$ then $P(A) = 0$. [3]

(ii) Show that $P(A^c) = 1 - P(A)$. [4]

(b) State and prove the Bayes' theorem. [5]

(c) A study on voting habits of married couples has shown that the probability that the husband votes on communal issues is 0.24, whereas the probability that the wife votes on such issues is 0.32 and the probability that both vote is 0.12.

(i) What is the probability that:

(a) either the husband or the wife will vote? [2]

(b) neither of them votes? [1]

(c) the wife votes knowing that the husband is going to vote? [2]

(ii) If the wife has decided to vote, what is the probability of the husband not voting? [3]

END OF QUESTION PAPER

Table of Laplace Transforms

	$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$		$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	$\mathbf{e}^{at}$	$\frac{1}{s-a}$
3.	$t^n,\quad n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	$t^p,\ p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}},\quad n=1,2,3,\ldots$	$\frac{1\cdot3\cdot5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$\mathbf{e}^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$\mathbf{e}^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$\mathbf{e}^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$\mathbf{e}^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n\mathbf{e}^{at},\quad n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$ <u>Heaviside Function</u>	$\frac{\mathbf{e}^{-cs}}{s}$	26.	$\delta(t-c)$ <u>Dirac Delta Function</u>	$\mathbf{e}^{-cs}$
27.	$u_c(t)f(t-c)$	$\mathbf{e}^{-cs}F(s)$	28.	$u_c(t)g(t)$	$\mathbf{e}^{-cs}\mathfrak{L}\{g(t+c)\}$
29.	$\mathbf{e}^{ct}f(t)$	$F(s-c)$	30.	$t^n f(t),\quad n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31.	$\frac{1}{t}f(t)$	$\int_s^\infty F(u)\,du$	32.	$\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\frac{\int_0^T \mathbf{e}^{-st}f(t)\,dt}{1-\mathbf{e}^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$			