NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA4103

FACULTY OF APPLIED SCIENCE

DEPARTMENT OF APPLIED MATHEMATICS

SMA4103: FLUID MECHANICS

AUGUST 2024 SPECIAL EXAMINATION

Time : 3 hours

Candidates should attempt **ALL** questions from **Section A** (40 marks) and **ANY THREE** questions from **Section B** (20 marks each).

SECTION A: Answer ALL questions in this section [40].

A1. Define the following:

(a) compressible fluid, [2]

(b) newtonian fluid, [2]

(c) particle path. [2]

A2. Compare briefly the Eulerian and Lagrangian descriptions of fluid motion. [3]

A3. An inviscid, incompressible, steady 2−D flow is given by $u = 4xy$, $v = -y^2$.

(a) Show that a stream function exits for this flow. [2]

(b) Find the stream function. [4]

(c) Plot and interpret the streamline pattern. [4]

A4. Air flows steadily and at low speed through a horizontal nozzle discharging to the atmosphere. At the nozzle inlet, the area is 0.2m^2 and at the nozzle outlet, the area is 0.04m^2. The flow is essentially incompressible, frictionless. Determine the pressure required at the nozzle inlet to produce an outlet speed of 100ms^{-1}. For air at standard conditions $\rho = 1.23\text{kgm}^{-3}$. [6]

A5. Using the theorem of Blasius which enables one to evaluate the resultant force and couple exerted on a unit length of a cylinder, by the moving fluid, show that the force exerted on the cylinder $|z| = a$ in the irrotational flow is given by $[X, Y]$ per unit length, where $X = 0$, $Y = 2\pi\rho U k$. [6]

A6. A flow is defined by $\mathbf{v} = (cy, -cx, 0)$.

(a) Find and describe the particle paths for the flow. [6]

(b) Sketch the particle paths. [2]

(c) What would be modelled by this type of flow. [1]

SECTION B: Answer THREE questions in this section [60].

B7. (a) For the basic complex potential flow

$$w(z) = c \ln z, \tag{1}$$

the type of flow is determined on whether the constant c is real or imaginary.

(i) Find the velocity potential and stream function for c imaginary. [3]

(ii) Find the velocity components for c imaginary. [4]

(iii) Find the strength of the flow. [3]

(iv) Describe the type of flow represented by (1). [2]

(b) (i) State Reynolds transport theorem. [3]

(ii) Use Reynolds transport theorem to derive the conservation of mass equation

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \vec{v} = 0.$$

[5]

B8. (a) Viscous fluid flows down a pipe of circular cross-section and radius $r = a$ under a constant pressure. $\vec{v} = (v_r, v_\theta, v_z) = u(r)\hat{\mathbf{e}}_\mathbf{z}$, from the Navier-Stokes equations show that for steady flow

$$\mu\left(\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr}\right) = \frac{dp}{dz} = -\frac{\Delta p}{L}, \quad v_r = v_\theta = 0.$$

[5]

(b) Show that the velocity profile is given by

$$u = \frac{\Delta p}{4\mu L}(a^2 - r^2), \quad v_r = v_\theta = 0.$$

[4]

(c) Sketch the fully developed laminar flow in the pipe. [2]

(d) Find the total volumetric flow rate. [4]

(e) If u_m is the mean velocity over the cross-section of the pipe, show that the pressure drop measured over the length L of the pipe is

$$\Delta p = \frac{32\mu u_m L}{D^2},$$

where D is the internal diameter of the pipe. [2]

(f) Derive an expression for the friction factor where the Reynolds number is less than 2000. [3]

B9. (a) (i) Derive Euler's equation of motion for the flow of an inviscid, incompressible fluid. [6]

(ii) If the flow is steady and subject to a conservative force $\mathbf{F} = -\nabla\Omega$ per unit mass, prove Bernoulli's theorem: $\frac{p}{\rho} + \frac{v^2}{2} + \Omega =$ constant, where p is the pressure, ρ is the uniform density and v is the fluid speed. [8]

(b) What is the physical significance of Bernoulli's theorem? [2]

(c) A large open tank is fitted with an outlet pipe of cross-sectional area A and length l which protrudes vertically downwards from the base of the tank. Water is put into the tank at a rate Q (volume per unit time) with negligible speed compared to the speed in the outlet pipe. Calculate the depth of the water in the steady state. [4]

B10. (a) In unidirectional flow of viscous incompressible fluid the velocity field $u(y,t)$ is found to satisfy:

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2},$$

where $P = P(x,t)$.

(i) Show that if the flow is steady, then $\frac{\partial P}{\partial x} = \frac{dP}{dx}$ is a constant and $\frac{\partial^2 u}{\partial y^2} = \frac{d^2 u}{dy^2}$ is also a constant. [3]

(ii) If the flow is steady and $\frac{dP}{dx} = -G$, where G is a positive constant, show that the flow velocity in the channel with impermeable walls $y = (0, h)$ is given by

$$u(y) = \frac{Gy}{2\mu}(h - y).$$

[4]

(iii) Sketch $u(y)$. In which direction is the flow and why? [4]

(b) The velocity field of an incompressible inviscid fluid is given by $u = -\eta y$, $v = \eta x$, $w = 0$, where η is a constant. Assuming that there are no body forces acting on the fluid,

(i) Show that the fluid vorticity is constant. [3]

(ii) Find the pressure at each point of the fluid. [6]

END OF QUESTION PAPER