



NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED PHYSICS

BSc (Hons) in Applied Physics PART 2

CLASSICAL MECHANICS

SPH 2103

First Semester Examination Paper

December 2024

This examination paper consists of 5 pages

Time Allowed: 3 hours

Total Marks: 100

Special Requirements: None

Examiner's Name: Mr T. Dube

INSTRUCTIONS

ANSWER ALL PARTS OF QUESTION 1 IN SECTION A AND ANY THREE QUESTIONS FROM SECTION B. SECTION A CARRIES 40 MARKS AND SECTION B CARRIES 60 MARKS.

MARK ALLOCATION

QUESTION	MARKS
1.	40
2.	20
3.	20
4.	20
5.	20
6.	20
Maximum possible mark	100

SECTION A

- 1 (a) Define inertial force and state two examples [4]
- (b) List two advantages of the Lagrangian approach to classical mechanics over the standard Newtonian approach? [2]
- (c) Consider the mechanical set up below in two dimensions. A bead of mass m_{bead} is attached to a massless rotating wheel of radius l_1 . The wheel rotates in a vertical plane with an angular velocity ω measured with respect to the horizontal. The end of a massless rod of length l_2 is attached to the bead and hangs vertically downwards from the bead. The rod is free to swing around the bead. As the rod moves, the angular displacement of the rod from the vertical line passing through the center of the bead is given by the angle θ . At the opposite end of the rod is a ball of mass m_{ball} . See the Fig 1 and answer the question that follow [4]

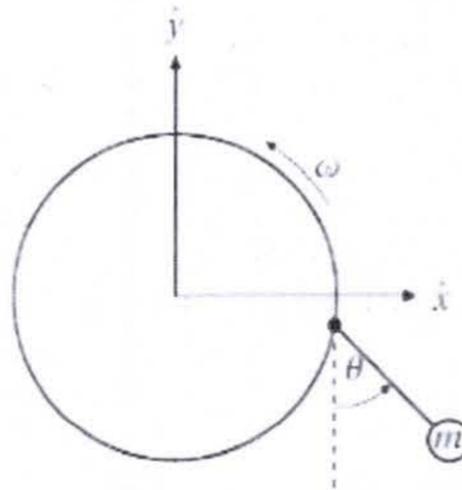


Fig 1

- (i) Determine the position of the bead at time t [2]
- (ii) Determine the position of the ball with respect to the bead at a time t in terms of the angular displacement θ . [2]
- (iii) Determine the position of the ball with respect to the origin. [2]
- (iv) Show that the kinetic energy of the bead is T_{bead} [2]

$$T_{\text{bead}} = \frac{1}{2} m_{\text{bead}} l_1^2 \omega^2$$
- Show that the kinetic energy of the ball is [5]
- (v) $T_{\text{ball}} = \frac{1}{2} m_{\text{ball}} (l_1^2 \omega^2 + l_2^2 \dot{\theta}^2 - 2l_1 l_2 \omega \sin(\omega t - \theta) \dot{\theta})$ [5]
- (vi) Determine the potential energy of the bead [2]

(vii) Determine the potential energy of the ball [3]

(viii) Give a Lagrangian for this system [2]

(d) Two light rods of length l_1 and l_2 are shown oscillating in the same plane as depicted in Fig 2. The rods are assumed to be massless and the bobs move under the influence of gravity.

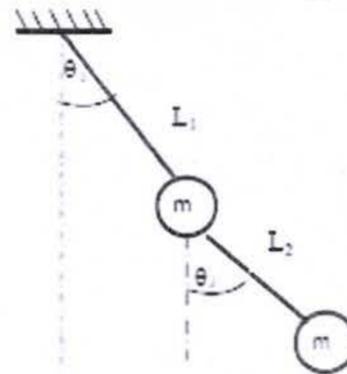


Fig 2

(i) Determine the number of degrees of freedom in this mechanical system. Explain your reasoning. [3]

(ii) Using the principles of classical mechanics, derive the Lagrangian for this system. Your response should include expressions for both the kinetic and potential energy of the system. [5]

(e) A particle of mass m is moving in one dimension, subject to a potential energy function $V(x)$. The kinetic energy of the particle is given by $T = \frac{1}{2}m\dot{x}^2$, where \dot{x} is the velocity of the particle. Express the Hamiltonian H of the system in terms of the particle's momentum p and the position x . [2]

SECTION B

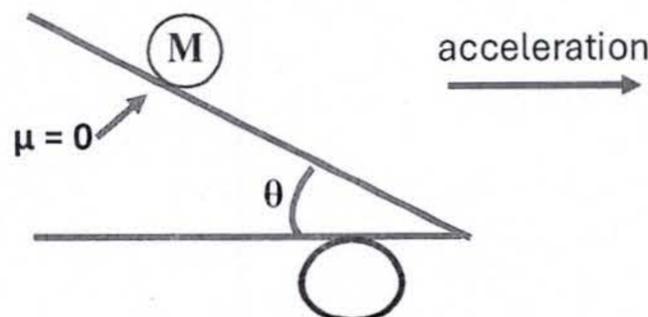
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Consider a two-body problem involving a planet of mass M and a satellite of mass m moving under the influence of their mutual gravitational attraction. The distance between the two bodies is denoted by r .

(a) Using the principles of Lagrangian mechanics, derive the expression for the Lagrangian L of the system. Include both kinetic and potential energy terms in your derivation. [8]

(b) Show how the conservation of energy is manifested in this system. Specifically, identify the conditions under which the total mechanical energy E (the sum of kinetic and potential energy) remains constant. [4]

- (c) Explain the significance of the conservation of energy in the context of orbital mechanics. How does this principle help us understand the motion of the satellite in its orbit around the planet? [4]
- (d) Discuss any assumptions made in your derivation and how they affect the results [4]
- 3 a) Identify and discuss at least four advantages of Hamiltonian mechanics over Lagrangian mechanics. [2x4]
- b) Identify and discuss at least three disadvantages of Hamiltonian mechanics when compared to Lagrangian mechanics. [2x3]
- c) Describe at least three specific scenarios or types of problems where the Hamiltonian formulation is particularly advantageous. Discuss why Hamiltonian mechanics is more beneficial in these cases. [6]
- 4 a) (i) Define an inertial reference frame. In your description, explain the fundamental characteristics that distinguish inertial frames from non-inertial frames. [5]
- (ii) Discuss how the laws of physics, particularly Newton's laws of motion, differ between these two types of frames [5]
- (b) (i) Explain the concept of fictitious forces and how they arise in non-inertial reference frames. In your explanation, include definitions, the significance of these forces in relation to Newton's laws of motion, and provide examples to illustrate your points. [5]
- (ii) Discuss the Coriolis force and the centrifugal force. For each force, include its mathematical expression and provide physical examples where each force is applicable. Discuss the implications of these forces in real-world scenarios. [5]
- 5 A bead of mass M sits on the sloped hood of a car. The hood is at an angle θ from the horizontal and the car is accelerating to the right as shown. Assume that the resistance on the bead and friction between the bead and the hood of the car are both negligible. As the car accelerates, the bead moves along with the car but it does not move up or down the slope of the hood.



- (a) Draw a free-body diagram for the bead showing all forces acting on it. [5]
- (b) pick a choice of X and Y axes, show them on your diagram, and write $F = ma$ equations for the two directions. [6]
- (c) Find the magnitude of the normal force of the hood on the bead and the acceleration of the car. Express your answers in terms of M , g and θ , noting that the result may depend on only one or two of these values. [9]
- 6 (a) Explain the principle of relativity as proposed by Einstein. How does it differ from classical mechanics in terms of space and time? Illustrate your explanation with appropriate examples [6]
- (b) A spaceship travels at a speed of $0.8c$ relative to Earth, where c is the speed of light. If 5 years pass for the astronauts on the spaceship, how much time has elapsed on Earth? Show your calculations and discuss the implications of your results. [7]
- (c) A rod measures 10 meters in its rest frame. If it moves at a speed of $0.6c$ relative to an observer, what is its length as measured by that observer? Provide the relevant equations and discuss the significance of length contraction in relativistic physics. [7]

END OF PAPER