NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCE

DEPARTMENT OF COMPUTER SCIENCE

DISCRETE MATHEMATICS

SCS 1210

EXAMINATION PAPER

March Intake Part I Second Semester Phase II 2024

This examination paper consists of 4 pages

Time Allowed: 3 hours

Total Marks: 100

Examiner's Name: Mr. S. Ngwenya

External Examiner: Dr C Gombiro

INSTRUCTIONS

1. Answer any four (4) questions
2. Each question carries 25 marks
3. Use of calculators is permissible

MARK ALLOCATION

QUESTION	MARKS
1.	25
2.	25
3.	25
4.	25
5.	25
TOTAL	100

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QUESTION ONE

a) Given that the number of elements in the power set of A is 4.

i) Write down the expression for the number of elements in the power set of A. [1]
ii) Determine the number of elements in set A. [2]
iii) Calculate the number of elements in AxA. [2]

b) P, Q and R are sets such that P = {5}, Q = {1} and R = ∅. List the elements of:
i) $P \cup R$ [1]
ii) $Q \cap R \cap P$ [2]

c) Given that M = {a, b, c} and N = {a}, list the elements of:

i) N – M [2]
ii) M + N [2]

d) Given that A, B and C are finite sets.
Prove that: $A \cap((C \cup B)' \cup(B' \cap C)) = A \cap B'$. [6]

e) Solve the recurrence relation $an = 2an-1 - 2an-2$ where n0 =1 and n2= 3
[7]

QUESTION TWO

a) Write the general expression for Permutations [3]

b) In how many ways can a photographer at a wedding arrange 6 people in a row from a group of 10 people (including a bride and a groom) if
1. The bride must be in the picture. [3]
2. The bride and the groom must be in the picture [3]
3. Exactly one of the bride or the groom must be in the picture [5]

c) The domain of discourse for x and y is the set of employees at a company. Define the predicate:
V(x): x is a manager
M(x, y): x earns more than y
Write the logical expression equivalent to: "Every manager earns more than every employee who is not a manager." [5]

d) Let A={1,2,3,4,5,6} and B={1,2,3,4}. Define (a,b)∈R if and only if (a−b) mod 2=0. [6]

QUESTION THREE

a) Consider a function f: Z → Z by the function f(x) =| x|. Determine,
   i) the domain of f(x). [2]
   ii) the co-domain of f(x). [3]
   iii) the range of f(x). [3]

b) Investigate if the function f: N → N given by $f(x) = x^2$ is an injective function or not. [3]

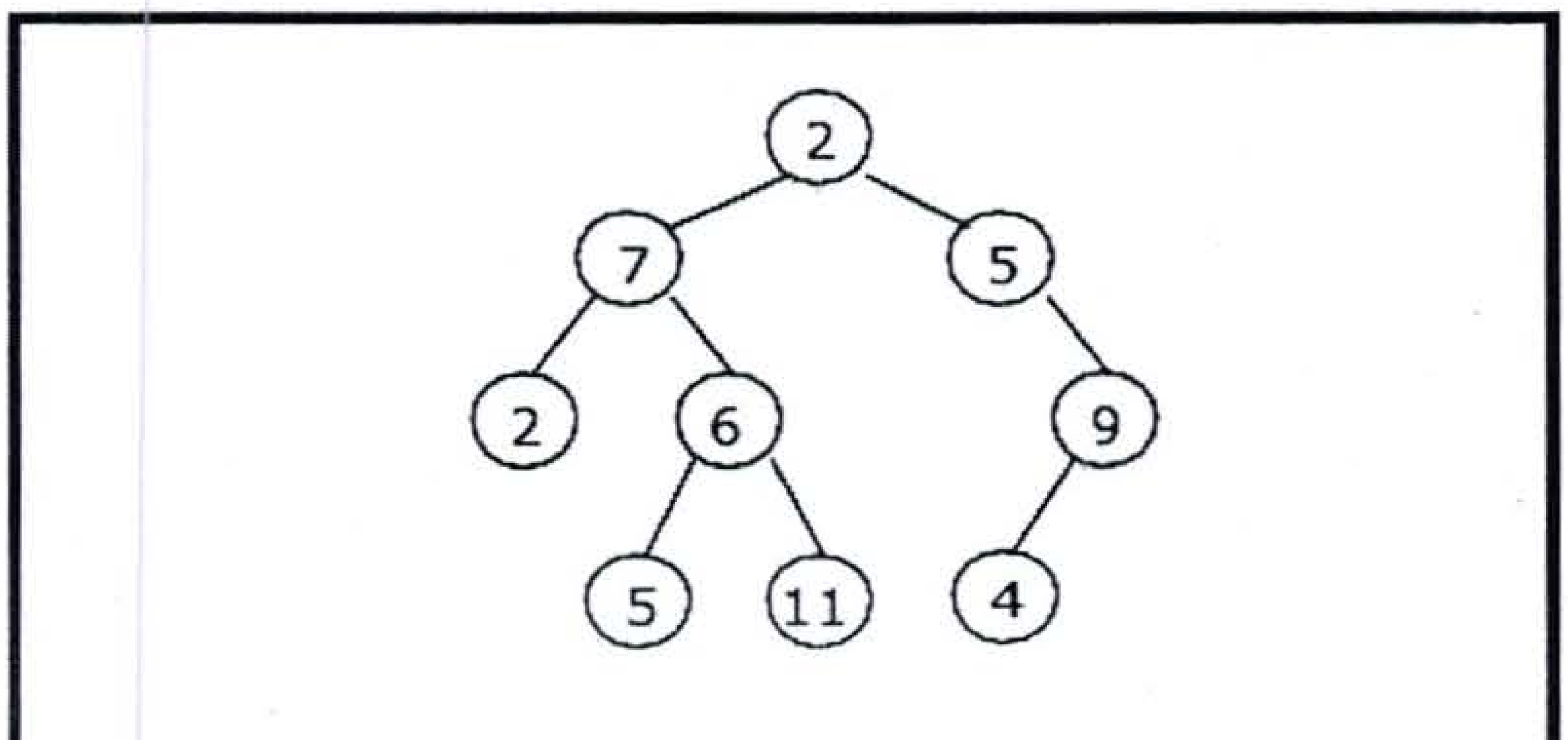
c) If the function f: R → R is given by $f(x) = x - 1$. Show that f(x) is an injective function. [4]

d) Given that f(x) = x – 8 and $g(x) = x^2 - x - 3$, find an expression for the following giving your answer in its simplest form:
   i) f(x) * g(x). [3]
   ii) ½ g(x) – ¾ f(x). [3]
   iii) $fog(x)^{-1}$. [4]

QUESTION FOUR

a) Write a general algorithm to traverse a non-empty binary tree in postorder [3]

b) Find the output of traversing the binary tree given below by inorder. [3]



c) Draw a binary tree where the nodes would be listed in preorder traversal as A B D C E F and determine the depth and tree level . [8]

d) Consider the following pairs of sentences. For each pair, determine if one implies the other if they are equivalent, or neither.

i. $\forall x \forall y P(x,y)$	$\forall y \forall x P(x,y)$	[3]
ii. $\exists x \exists y P(x,y)$	$\exists y \exists x P(x,y)$	[3]
iii. $\forall x \exists y P(x,y)$	$\forall y \exists x P(x,y)$	[5]

QUESTION FIVE

a) Suppose that in a group of 5 people: A, B, C, D, and E, the following pairs of people are acquainted with each other.
A and C
A and D
B and C
C and D
C and E
i. Draw a graph G to represent this situation. [2]
ii. List the vertex set, and the edge set, using set notation. In other words, show sets V and E for the vertices and edges, in G = {V, E}. [3]

b) Consider a simple undirected graph of 10 vertices. Find the maximum number of edges in the graph if it is disconnected. [5]

c) Indicate whether the following statements are tautology, contradiction, or neither.
i $A \leftrightarrow (A \vee A)$ [5]
ii $(A \vee B) \rightarrow B$
[5]
iii $A \vee (\neg(A \vee B))$ [5]

END QUESTION PAPER