NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCE

DEPARTMENT OF COMPUTER SCIENCE

COMPUTATIONAL DISCRETE MATHEMATICS

SCS 5102

EXAMINATION PAPER

FIRST SEMESTER 2024

This examination paper consists of 5 pages

Time Allowed: 3 hours

Total Marks: 100

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External Examiner: Dr C. Gombiro

INSTRUCTIONS

1. Answer any four (4) questions.
2. Each question carries 25 marks
3. Use of calculators is permissible

MARK ALLOCATION

QUESTION	MARKS
1.	**25**
2.	**25**
3.	**25**
4.	**25**
5.	**25**
TOTAL	**100**

QUESTION ONE

(a) Let p: Jupiter is a planet and q: India is an island be any two simple statements. Give a verbal sentence describing each of the following statements.

(i) $\neg p$ [1]

(ii) $p \vee \neg q$ [2]

(iii) $\neg p \vee q$ [2]

(iv) $p \rightarrow \neg q$ (v) $p \leftrightarrow q$ *[2]*

(b) Construct the truth table for the following statements.

(i) $\neg p \wedge \neg q$ [2]

(ii) $\neg (p \wedge \neg q)$ [2]

(iii) $(p \vee q) \neg q$ (iv) $(\neg p \rightarrow r) \wedge (p \leftrightarrow q)$ [4]

(c) Using truth table check whether the statements $\neg (p \vee q) \vee (\neg p \wedge q)$ and $\neg p$ are logically equivalent. [5]

(d) Prove $p \rightarrow (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$ without using truth table. [5]

QUESTION TWO

a. Consider selecting 4 objects from Set A= {1,2,3,4,5,6,9,10,12,14,15}

I. How many ordered sequences can be chosen from A? [2]

II. How many ordered sequences with repetition can be chosen? [2]

III. How many unordered sequences with repetition can be chosen from A? [3]

IV. How many unordered sequences without repetition can be chosen from A? [2]

V. How many strictly decreasing sequences can be chosen from A? [3]

b. Let g: A → B and f: B→ C be two functions. Demonstrate that if g and f are both injective, then $f \circ g$: A →C is injective. [7]

c. Let f: A→ B and g: B→ C. Show that if $g \circ f$: A→ C is surjective, then g is surjective. [6]

QUESTION THREE

a. The Permutation P of 6 objects taking x at a time = 360.

I. Write the expressions for Permutation P and Combination C. [4]

II. Determine the value of x. [5]

b. Consider a basket with 5 apples and 4 pears.

I. In how many ways can you collect three apples and two pears? [3]

II. How many different equivalence relations with three equivalence classes are there on the set {1,2,3,4,5}? [7]

III. How many surjective functions are there from the set {1, 2, 3, 4} or {1, 2, 3}? [6]

QUESTION FOUR

a. Draw the following graphs or explain why they cannot exist.

I. A graph with an isolated vertex and a universal vertex [3]

II. A cubic graph of order 5. [3]

III. A bipartite graph of order 5 and size 7. [3]

IV. A bipartite graph of order 8 and size 10. [3]

(b) Using graph theory, explain whether it is possible for each person, in a group of 15 individuals, to have exactly three friends. (Assume that friendship is a symmetric relation, i.e., friendship goes both ways.) [6]

(c) Prove that if G is a graph with n vertices and n edges with no vertices of degree 0 or 1, then the degree of every vertex is 2. [7]

QUESTION FIVE

a. Draw the state diagrams for the finite-state machine with this state table in figure 5.1. [8]

State	f		g	
	Input		Input	
	0	1	0	1
S_0	S_1	s_0	0	1
S_1	S_0	s_2	0	1
S_2	S_1	s_1	0	0

Figure 5.1: State Table

b. Draw a state diagram for a traffic light controller. Indicate the logical conditions for remaining in the current state and for moving to the next state. Please number your states 0 to n and label outputs with meaningful names such as 'B Green", 'B Yellow", 'A South Green Arrow", 'A North Red" etc [10]

c. Create a table that indicates precisely which lights are illuminated for each of your states. Use the following abbreviations: R = Red, Y = Yellow, G = Green, GA = Green left turn arrow, YA = Yellow left turn arrow, and indicate the direction and street. [7]

END QUESTION PAPER