NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SBA1201

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF STATISTICS AND OPERATIONS RESEARCH

SBA1201: INTRODUCTION TO LINEAR PROGRAMMING

BSc. Business Analytics: Part I

APRIL 2025 EXAMINATION

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B. Each question should start on a fresh page. Graph paper required.

SECTION A: Answer all questions in this section (40 marks).

A1. (a) Define linear programming. [2]

(b) State and briefly explain the assumptions of linear programming. [8]

(c) State any two advantages and disadvantages of linear programming models. [4]

A2. (a) What is an optimal solution as used in linear programming? [4]

(b) Discuss the differences between feasible and optimal solutions. [4]

(c) A farmer has 1,000 acres of land on which he can grow corn, wheat and soya beans. Each acre of corn costs \$100.00 for preparations, requires 7 man-days of work and yields a profit of \$30.00. An acre of wheat costs \$120.00 to prepare, requires 10 man-days of work and yields a profit of \$40.00. An acre of soya beans costs \$70.00 to prepare, requires 8 man-days of work and yields a profit of \$20.00. If the farmer has \$100 000.00 for preparation and can count on 8000 man-days of work, formulate the linear programming model to allocate the number of acres to each crop to maximise the total profit. [8]

A3. (a) Explain the terms infeasibility, unbounded and multiple optimal solutions. [3]

(b) Does the following linear programming problem exhibit infeasibility, unboundedness, or alternate optimal solutions? Explain. [7]

$$\begin{aligned} \text{Min } Z &= x_1 + x_2 \\ &\text{subject to} \\ 5x_1 + 3x_2 &\leq 30 \\ 3x_1 + 4x_2 &\geq 36 \\ x_2 &\leq 7 \\ \text{and } x_i &\geq 0, i = 1, 2 \end{aligned}$$

SECTION B: Answer any three questions in this section (60 marks).

B4. This is your lucky day. You have just won $10,000 prize. You are setting aside $4,000 for taxes and partying expenses, but you have decided to invest the other $6,000. Upon hearing this news, two different friends have offered you an opportunity to become a partner in two different entrepreneurial ventures, one planned by each friend. In both cases, this investment would involve expending some of your next time summer as well as putting up cash. Becoming a full partner in the first friend's venture would require an investment of $5,000 and 400 hours, and your estimated profit (ignoring the value of your time) would be $4,500. The corresponding figures for the second's friend venture are $4,000 and 500 hours, with an estimated profit to you of $4,500. However, both friends are flexible and would allow you to come in at any fraction of a full partnership, all the above figures given for full partnership (money investment, time investment, and your profit) would be multiplied by this same fraction.

Because you were looking for an interesting summer job anyway (maximum of 600 hours), you have decided to participate in one or both friends' ventures in whichever combination would maximise your total estimated profit. You now need to solve the problem of finding the best combination.

(a) Describe the analogy in this problem, identifying both the activities and resources. [8]

(b) Formulate a linear programming model for this problem. [6]

(c) Use the graphical method to solve this model. What is your total estimated profit? [6]

B5. Consider the following problem

$$\text{Max } Z = 2x_1 + 6x_2 + 9x_3$$
$$\text{subject to}$$
$$x_1 + x_3 \leq 3 (\text{resource } 1)$$
$$x_2 + 2x_3 \leq 5 (\text{resource } 2)$$
$$\text{and } x_i \geq 0, i = 1, 2, 3$$

(a) Construct the dual for this primal problem. [4]

(b) Solve the dual problem graphically. Use this solution to identify the shadow prices for the resources in the primal problem. [6]

(c) Confirm your results from part (*b*) by solving the primal problem by simplex method and then identify the shadow prices. [10]

B6. (a) Use the two-phase simplex method to

$$\text{Max } Z = 2x_1 + x_2 + \tfrac{1}{4}x_3$$
$$\text{subject to}$$
$$4x_1 + 6x_2 + 3x_3 \leq 8$$
$$3x_1 - 6x_2 - 4x_3 \leq 1$$
$$2x_1 + 3x_2 - 5x_3 \geq 4$$
$$\text{and } x_i \geq 0, i = 1, 2, 3$$

[15]

(b) Consider the following linear programming problem

$$\text{Min } Z = 2x_1 + 3x_2$$
$$\text{subject to}$$
$$x_1 - 2x_2 \leq 0$$
$$-2x_1 + 3x_2 \geq -6$$
$$2x_1 + 3x_2 - 5x_3 \geq 4$$
$$\text{and } x_1, x_2 \textit{ unrestricted}$$

(i) Setup the problem in the standard form. [4]

(ii) Find the initial basic feasible solution. [1]

B7. Solve the following Linear Programming

$$\text{Max } Z = 4x_1 + 10x_2$$
$$\text{subject to}$$
$$2x_1 + x_2 \leq 10$$
$$2x_1 + 5x_2 \leq 20$$
$$2x_1 + 3x_2 \leq 18$$
$$\text{and } x_i \geq 0, i = 1, 2$$

(a) Indicate that this problem has an alternative optimal basic feasible solution. [10]

(b) Find the optimal solution. [6]

(c) Hence show that this problem has multiple optimal solutions. [4]

END OF QUESTION PAPER