

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF STATISTICS AND OPERATIONS RESEARCH

SBA1201: INTRODUCTION TO LINEAR PROGRAMMING

BSc. BUSINESS ANALYTICS: PART I

SEPTEMBER 2024 EXAMINATION

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B. Each question should start on a fresh page. Graph paper required.

SECTION A: Answer all questions in this section (40 marks).

A1. Define the following terms:

- (a) Objective function [2]
- (b) Constraint [2]
- (c) Binding constraint [2]
- (d) Feasible solution [2]
- (e) Psuedo optimal solution [2]

- A2.** (a) State and briefly explain the assumptions of linear programming. [8]
- (b) A farmer has 1,000 acres of land on which he can grow corn, wheat and soya beans. Each acre of corn costs ZWL1000.00 for preparations, requires 7 man-days of work and yields a profit of ZWL300.00. An acre of wheat costs ZWL1200.00 to prepare, requires 10 man-days of work and yields a profit of ZWL400.00. An acre of soya beans costs ZWL700.00 to prepare, requires 8 man-days of work and yields a profit of ZWL200.00. If the farmer has ZWL1 000 000.00 for preparation and can count on 8000 man-days of work, formulate the linear programming model to allocate the number of acres to each crop to maximise the total profit. [8]

A3. Given the linear programming problem

$$\begin{aligned} \text{Max } Z &= 3x_1 + 2x_2 \\ \text{subject to} \\ x_1 + 2x_2 &\leq 12 \\ 2x_1 + 3x_2 &= 12 \\ 2x_1 + x_2 &\geq 8 \\ \text{and } x_i &\geq 0, i = 1, 2 \end{aligned}$$

- (a) Write the problem in standard form. [3]
 (b) Write the problem in canonical form. [3]
 (c) Solve the problem graphically. [8]

SECTION B: Answer any three questions in this section (60 marks).

- B4. (a) The Primeworld Company produces two light fixtures (products 1 and 2) that require both metal frame parts and electrical components. Management wants to determine how many units of each product to produce so as to maximize profit. For each unit of product 1, 1 unit of frame parts and 2 units of electrical components are required. For each unit of product 2, 3 units of frame parts and 2 units of electrical components are required. The company has 200 units of frame parts and 300 units of electrical components. Each unit of product 1 gives a profit of \$1, and each unit of product 2, up to 60 units, gives a profit of \$2. Any excess over 60 units of product 2 brings no profit, so such an excess has been ruled out.
- (i) Formulate a linear programming model for this problem. [5]
 (ii) Use the graphical method to solve this model. What is the resulting total profit? [8]
- (b) Consider the objective function Maximize $Z = 2x_1 + 3x_2$ for a linear programming model:
- (i) Draw a graph that shows the corresponding objective function lines for $Z = 6$, $Z = 12$, and $Z = 18$. [4]
 (ii) Find the slope-intercept form of the equation for each of these three objective function lines. Compare the slope for these three lines. Also compare the intercept with the x_2 axis. [3]

B5. Consider the following problem

$$\begin{aligned} \text{Max } Z &= 2x_1 + 6x_2 + 9x_3 \\ &\text{subject to} \\ x_1 + x_3 &\leq 3(\text{resource 1}) \\ x_2 + 2x_3 &\leq 5(\text{resource 2}) \\ \text{and } x_i &\geq 0, i = 1, 2, 3 \end{aligned}$$

- (a) Construct the dual for this primal problem. [4]
 (b) Solve the dual problem graphically. Use this solution to identify the shadow prices for the resources in the primal problem. [6]
 (c) Confirm your results from part (b) by solving the primal problem by simplex method and then identify the shadow prices. [10]

B6. Waffles contains 6 units of calories per gram and 7 units of vitamin B per gram and costs 12 zig per gram. Condensed milk contains 8 units of calories per gram and 12 units of vitamin B per gram and costs 20 zig per gram. The daily minimum requirement of calories and vitamin B is 100 units and 120 units respectively. Find the minimum cost of product mix by the simplex method. [20]

B7. (a) Use the two-phase simplex method to

$$\begin{aligned} \text{Max } Z &= 2x_1 + x_2 + \frac{1}{4}x_3 \\ &\text{subject to} \\ 4x_1 + 6x_2 + 3x_3 &\leq 8 \\ 3x_1 - 6x_2 - 4x_3 &\leq 1 \\ 2x_1 + 3x_2 - 5x_3 &\geq 4 \\ \text{and } x_i &\geq 0, i = 1, 2, 3 \end{aligned}$$

[15]

(b) Consider the following linear programming problem

$$\begin{aligned} \text{Min } Z &= 2x_1 + 3x_2 \\ &\text{subject to} \\ x_1 - 2x_2 &\leq 0 \\ -2x_1 + 3x_2 &\geq -6 \\ 2x_1 + 3x_2 - 5x_3 &\geq 4 \\ \text{and } x_1, x_2 &\text{ unrestricted} \end{aligned}$$

- (i) Setup the problem in the standard form. [4]
 (ii) Find the initial basic feasible solution. [1]

END OF QUESTION PAPER