

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF STATISTICS AND OPERATIONS RESEARCH

SBA2204: STOCHASTIC OPTIMISATION

MARCH 2025: EXAMINATION

Time : 3 hours

Candidates should attempt **ALL** questions from **Section A** (40 marks) and **ANY THREE** questions from **Section B** (20 marks each).

SECTION A

Answer **ALL** questions in this section [40 Marks]

- A1.** (a) Define a stochastic process and explain its significance in optimization. [6]
(b) Differentiate between discrete-time and continuous-time Markov chains. [6]
(c) Describe the key steps involved in conducting a Monte Carlo simulation. [6]
(d) Explain the concept of a random walk and its application in financial modelling [6]

- A2.** A Markov chain has three states: X, Y, and Z. The transition probabilities are as follows:

$$\begin{aligned} X \rightarrow Y : 0.5, & \quad X \rightarrow Z : 0.5 \\ Y \rightarrow X : 0.4, & \quad Y \rightarrow Z : 0.6 \\ Z \rightarrow X : 0.1, & \quad Z \rightarrow Y : 0.9 \end{aligned}$$

- (a) What is meant by steady-state distribution in a Markov chain [4]
(b) Determine whether this system will reach a steady-state distribution and calculate the probability of transitioning from X to Z via Y. [12]

SECTION B

Answer any THREE questions in this section [60 Marks]

- B3.** A logistics company models its delivery process using a Markov Chain with the following transition probabilities between states: On-Time, Delayed, and Lost.
- On-Time to Delayed: 0.3
 - On-Time to Lost: 0.1
 - Delayed to On-Time: 0.5
 - Delayed to Lost: 0.2
 - Lost to On-Time: 0.4
 - Lost to Delayed: 0.6
- (a) Identify the states and represent the transitions in a state diagram. [4]
- (b) Construct the transition probability matrix for the system? [8]
- (c) Determine if there are any absorbing states or recurrent patterns and discuss the implications for the logistics process. [8]
- B4.** A tech startup is using Stochastic Gradient Descent (SGD) to optimize the performance of its recommendation algorithm. The dataset contains 50,000 user interactions.
- (a) Outline the process of implementing SGD, highlighting the roles of the learning rate and batch size. [10]
- (b) Discuss one common issue faced when using SGD and propose a mitigation method. [7]
- (c) Name and briefly describe three optimization algorithms that are variants of SGD. [3]
- B5.** A financial analyst wants to assess the risk of a new investment portfolio using Monte Carlo simulation. The following are key variables.
- Annual return: Normally distributed with a mean of 8% and a standard deviation of 3%.
 - Market volatility: Uniform distribution between 10% and 20%.
 - Economic downturn probability: 20%, causing a potential loss of 15% of the portfolio value.
- (a) Describe how the analyst should set up and conduct the Monte Carlo simulation. [12]
- (b) Explain how the results of this simulation can aid in making informed investment decisions. [8]

B6. A farmer uses stochastic dynamic programming to decide crop planting under uncertain rainfall. The states are Dry, Moderate, and Wet, with transition probabilities as follows:

Dry \rightarrow Dry (0.7), Moderate (0.2), Wet (0.1)

Moderate \rightarrow Dry (0.4), Moderate (0.4), Wet (0.2)

Wet \rightarrow Dry (0.2), Moderate (0.3), Wet (0.5)

- (a) What is the type of this chain , explain your answer. [5]
- (b) Present the transition probabilities in a transition probability matrix. [3]
- (c) Identify the components of Markov Chain based on the information above. [7]
- (d) Discuss how this model helps the farmer mitigate climate risks. [5]

END OF QUESTION PAPER