

NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY
SBA4101

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF STATISTICS AND OPERATIONS RESEARCH

SBA4101: NON LINEAR OPTIMISING AND DYNAMIC PROGRAMMING

BSc. BUSINESS ANALYTICS: PART 4

MARCH 2025 SPECIAL SUPPLEMENTARY EXAMINATION

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B. Each question should start on a fresh page. Graph paper required.

SECTION A: Answer all questions in this section (40 marks).

- A1.** (a) Define the following with an aid of a diagram:
- (i) Local maximum, [2]
 - (ii) Local minimum. [2]
- (b) How do you find concavity and convexity of a function? [6]
- A2.** For each of the following functions show whether it is convex, concave or neither:
- (a) $f(x) = 10 - x^2$. [2]
 - (b) $f(x) = x^4 + 6x^2 + 12x$. [2]
 - (c) $f(X) = x_1x_2 - x_1^2 - x_2^2$. [3]
 - (d) $f(x) = x^4 + x^2$. [3]
 - (e) $f(X) = x_1x_2$. [3]
- A3.** Show that $(0, 0)$ is a saddle point to the function $f(x_1, x_2) = 18x_1x_2 + 5x_2^2$. [7]

A4. $60m^3$ of a granular product are to be transported across a river in a ferry. The transported cost across the river is \$100 per trip, irrespective of the amount transported, but there is restriction on the number of trips, which should not be more than 40. The cost of the container depends upon its dimension as given below:

cost of bottom = \$10 per square metre

cost of front back sides = \$10 per square metre

cost of the ends = \$20 per square metre

Thus if a small container is used, cost of container will be less, but the number of trips will be more, and if a large container is used, cost of container will be more, but number of trips will be less. Find the dimensions of the container so that the sum of the container cost and transportation cost is minimized. [10]

SECTION B: Answer any three questions in this section (60 marks).

B5. (a) Consider the following Non Linear Programming Problem:

$$\text{Minimise } Z = 2x_1^2 - 24x_1 + 2x_2^2 - 8x_2 + 2x_3^2 - 12x_3 + 200.$$

By separating this function into three one-variable functions, show that the function is convex. Solve the problem by solving each one-variable function by calculus. [10]

(b) Solve the following Non Linear Programming Problem:

$$\begin{aligned} \text{Optimise } Z &= 4x_1 + 9x_2 - x_1^2 - x_2^2 \\ \text{subject to} \\ 4x_1 + 3x_2 &= 15, \\ 3x_1 + 5x_2 &= 14, \\ x_1, x_2 &\geq 0. \end{aligned}$$

[10]

B6. A company has divided its marketing area into three regions. The amount of sales depends upon the number of salesmen in each region. The company has been collecting the data regarding sales and salesmen in each area over a number of past years. The information is summarised in Table 1 below. For the next year, the company has only 9 salesmen and the problem is to allocate these salesmen to three regions so that the total sales are maximum.

Table 1: Profits in thousands of ZWG

No. of salesmen	Region 1	Region 2	Region 3
0	30	35	42
1	45	45	54
2	60	52	60
3	70	64	70
4	79	72	82
5	90	82	95
6	98	93	102
7	105	98	110
8	100	100	110
9	90	100	110

[20]

- B7.** (a) Solve the programming problem with lower and upper bounds of 0 and 2 respectively and an error tolerance $\epsilon = 0.01$:

$$\text{Minimise } Z = x^4 - x^2 - 4x. \quad [10]$$

- (b) Determine graphically the values of x_1 and x_2 so as to

$$\begin{aligned} &\text{Maximize } Z = 2x_1 + 3x_2, \\ &\text{subject to} \\ &\quad x_1x_2 \leq 8, \\ &\quad x_1^2 + x_2^2 \geq 20, \\ &\quad x_1, x_2 \geq 0. \end{aligned}$$

Verify that the Kuhn-Tucker conditions hold for the maxima. [10]

- B8.** Show three iterations of the solution for the following two-variable concave function $f(X)$ using the gradient search procedure.

$$\text{Maximise } f(X) = 2x_1x_2 + 2x_2 - x_1^2 - 2x_2^2.$$

$X = (\frac{1}{2}, \frac{1}{2})$ may be taken as the starting and trial solution. Draw the path of the trial solutions by solving the systems of linear equations by setting $\Delta f(X) = 0$. [20]

END OF QUESTION PAPER