

FACULTY OF APPLIED SCIENCE

DEPARTMENT OF STATISTICS AND OPERATIONS RESEARCH

INTRODUCTION TO OPERATIONS RESEARCH

BSc. OPERATIONS RESEARCH & STATISTICS: PART I

DECEMBER 2024 EXAMINATION

Time : 3 hours

Total Marks: 100

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B. Each question should start on a fresh page.

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**SECTION A: Attempt all questions in this section (40 marks).**

- A1.** State any three assumptions of Linear Programming. [3]
- A2.** Define each of the following terms:
- (a) Operations Research. [2]
  - (a) Optimum solution. [2]
  - (b) Slack variable. [2]
  - (c) Redundant constraint. [2]
  - (d) Shadow price. [2]

- A3. (a) State the five different ways in which models are classified. [5]
- (b) What is the difference between:
- (i) Deterministic models and Probabilistic models. [2]
- (ii) Static models and Dynamic models. [2]

A4. Figure 1 is a graphical solution from a Linear Programming Problem.

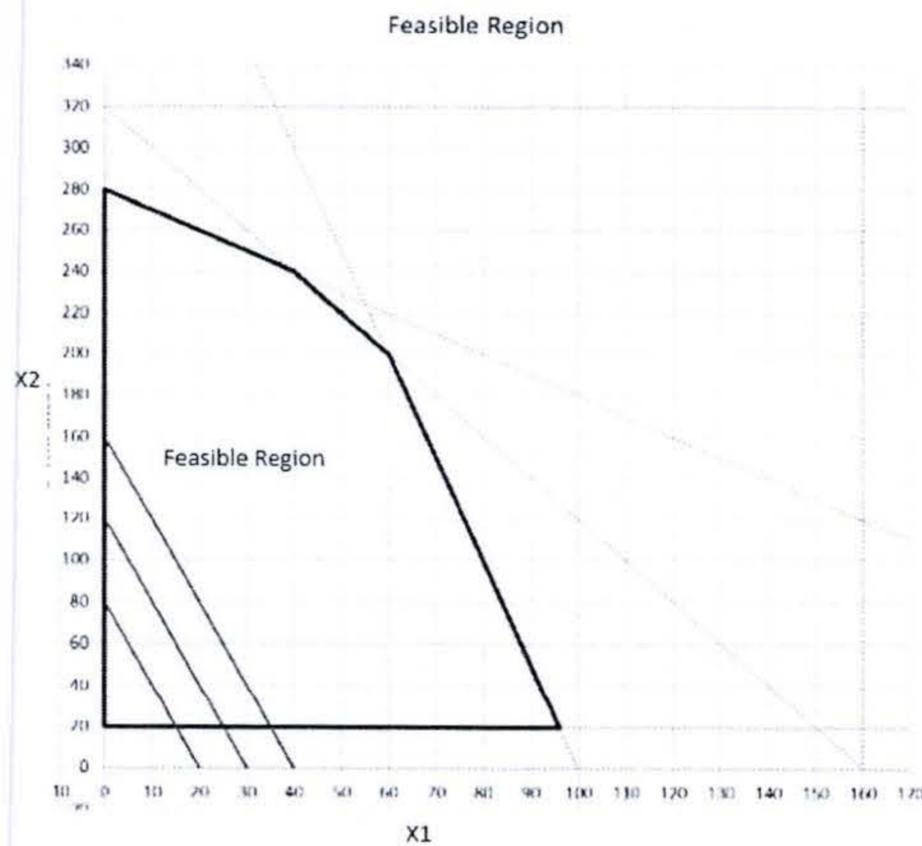


Figure 1: Graphical Solution

- (a) List all the constraints defined by the feasible region. [8]
- (b) Given the Objective function:

$$\text{Max } Z = 80x_1 + 20x_2,$$

Determine the optimal solution. [2]

A5. Use the graphical method to find the optimal solution(s) for the following model:

$$\text{Max } Z = 6x_1 + 9x_2$$

subject to

$$20x_1 + 40x_2 \leq 30000$$

$$30x_1 + 10x_2 \leq 30000$$

$$20x_1 + 16x_2 \leq 30000$$

$$x_1, x_2 \geq 0$$

[8]

**SECTION B: Attempt any three questions in this section (60 marks).**

- B6.** Expedition Outfitters manufactures a variety of speciality clothing for hiking, skiing, and mountain climbing. Its management decided to begin production on two new parkas designed for use in extremely cold weather: the Mount Everest Parka and the Rocky Mountain Parka. The manufacturing plant has 120 hours of cutting time and 120 hours of sewing time available for producing these two parkas. Each Mount Everest Parka requires 30 minutes of cutting time and 45 minutes of sewing time, and each Rocky Mountain Parka requires 20 minutes of cutting time and 15 minutes of sewing time. The labor and material cost is \$150 for each Mount Everest Parka and \$50 for each Rocky Mountain Parka, and the retail prices through the firms mail order catalog are \$250 for the Mount Everest Parka and \$200 for the Rocky Mountain Parka. Because management believes that the Mount Everest Parka is a unique coat that will enhance the image of the firm, they specified that at least 20% of the total production must consist of this model. Assuming that Expedition Outfitters can sell as many coats of each type as it can produce.
- (a) Formulate a linear programming problem that determine the number of units of each model that the organization should produce. [10]
  - (b) Solve this problem graphically to determine the maximum profit, showing all your working. [10]
- B7.** With the aid of well annotated graphs, describe the following special cases of solutions to a Linear Programming Problem.
- (a) Unbounded Solutions
  - (b) Degenerate Optimal Solutions
  - (c) Alternative Optima
  - (d) Infeasible Solutions

[20]

B8. Figure 2 shows the output of a linear programming solver.

Optimal Objective Value = 7668.00000			
Variable	Value	Reduced Cost	
S	540.00000	0.00000	
D	252.00000	0.00000	
Constraint	Slack/Surplus	Dual Value	
1	0.00000	4.37500	
2	120.00000	0.00000	
3	0.00000	6.93750	
4	18.00000	0.00000	
Variable	Objective Coefficient	Allowable Increase	Allowable Decrease
S	10.00000	3.50000	3.70000
D	9.00000	5.28571	2.33333
Constraint	RHS Value	Allowable Increase	Allowable Decrease
1	630.00000	52.36364	134.40000
2	600.00000	Infinite	120.00000
3	708.00000	192.00000	128.00000
4	135.00000	Infinite	18.00000

Figure 2: Computer Output

- (a) What is the optimal solution and the value associated with this solution? [3]
- (b) Which constraints are binding? [2]
- (c) What are the dual values for the constraints? Interpret each. [4]
- (d) Determine the objective coefficient ranges. [2]
- (e) Interpret the right hand side ranges. [4]
- (f) Will the optimal solution change if the profit margins are reversed? How much profit will be earned then? [3]
- (g) Suppose the objective coefficient of variable D increases from its current value of 9 to 11. How does the optimal solution change, if at all? [2]

- B9. A certain chemical manufacturer produces three types of chemicals. The company has been contracted to supply the following amounts of the three chemicals:

Table 1: Amounts of chemical supplied

Chemical	Contracted Sales (kg)
1	2000
2	3500
3	1800

The company is limited by the availability of processing time in two chemical reactors. Each chemical must be processed first in reactor 1 and then in reactor 2. Table 2 provides the hours of processing time available next month for each reactor and the processing time in each reactor per kg for each chemical.

Table 2: Allocation of Resources

Type of Reactor	1	2	3	Reactor Availabilities
Reactor 1	0.05	0.04	0.01	200hours
Reactor 2	0.02	0.06	0.03	150hours

Owing to the limited availability of reactor processing time the company has insufficient capacity to meet its demand with in-house production. Consequently, the company must purchase some chemicals from vendors having excess capacity and resell them to its own customers. Table 3 provides each chemical's in-house production cost and outside purchase cost.

Table 3: In-house production cost and Outside purchase cost

Chemical	In-house Production Cost (\$/kg)	Outside Purchase Cost (\$/kg)
1	2.50	2.80
2	1.75	2.50
3	2.90	3.25

The company's objective is to fill its customers' orders with the cheapest combination of in-house production and outside purchases. In short the company must decide how much of each chemical to produce in-house and to purchase outside.

[20]

END OF QUESTION PAPER