NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SORS 2101

FACULTY OF APPLIED SCIENCE

DEPARTMENT OF STATISTICS AND OPERATIONS RESEARCH

TIME SERIES ANALYSIS

BSc. Operations Research & Statistics: Part II

DECEMBER 2024 EXAMINATION

Time : 3 hours

Total Marks: 100

Candidates may attempt **ALL** Questions in **Section A** and at most **THREE** Questions in **Section B**. For all questions where necessary clearly show your work to indicate how you obtained the answer. Throughout this paper a_t represents white noise $E(a_t) = 0$ and $E(a_t^2) = \sigma^2$.

SECTION A: Attempt all questions in this section (40 marks).

A1. Define the term time series. [2]

A2. Explain what you understand by the following terms/concepts as applied to time series:

(a) Stochastic process. [2]

(b) White noise process. [2]

(c) Weak stationarity. [2]

A3. Consider the following MA(2) model

$$Z_t = (1 - 0.7B + 0.5B^2)a_t$$

(a) Determine whether the process is invertible. [4]

(b) Find the a.c.f of the MA(2) process. [6]

A4. A process is defined by $Z_t = a_t - 0.3a_{t-1} + 0.7a_{t-2} + 0.4a_{t-3}$, $a_t \approx N(0, \sigma^2)$. Calculate the:

(a) mean of Z_t, [2]

(b) variance of Z_t, [2]

(c) autocovariance of Z_t, [3]

(d) autocorrelation of Z_t. [3]

A5. (a) Obtain the model equations of the following time series in terms of θ, Θ, ϕ and Φ.

(i) SARIMA $(0, 0, 0) \times (0, 0, 1)_4$ [2]

(ii) SARIMA $(1, 0, 0) \times (1, 0, 1)_{12}$ [2]

(b) A series of data follows an ARIMA model (1,1,1) with $\phi_1 = 0.3$, $\theta_1 = -0.2$. Calculate the forecast for t=101 and t=102 given that $Z_{99} = 80$, $Z_{100} = 90$, and $f_{100} = 87$. Assume the ARIMA(1,1,1) to be a model with constant c=4.0. [8]

SECTION B: Attempt any three questions in this section (60 marks).

B6. The quarterly earnings of a small to medium soft drink company have been recorded for the years 2020-2023. The data (in $000 000) is shown in Table 1:

Table 1: Quarterly Earnings

Year	$Q1$	$Q2$	$Q3$	$Q4$
2020	6.37	5.55	6.16	6.41
2021	7.38	7.24	8.55	9.11
2022	9.67	8.06	8.91	7.82
2023	7.17	5.76	5.78	5.14

(a) Produce a time series plot for this data set and comment. [3]

(b) Calculate a 4−point moving averages and the seasonal ratios for this time series. [5]

(c) Plot the original time series and the moving average values on the same graph and comment. [4]

(d) Calculate the seasonal indices. [4]

(e) Deseasonalise these data using the seasonal indices obtained in (d). [4]

B7. (a) State and describe the components of time series. [6]

(b) For an AR(2) process

(i) Find conditions for stationarity. [4]

(ii)Find the ACF, up to lag 4. [6]

(iii) Calculate the ACF for $\phi_1 = 0.1$ and $\phi_2 = -0.3$. [4]

B8. (a) Consider an ARMA process given by:

$$Z_t = 0.5Z_{t-1} + a_t - 0.3a_{t-1}$$

(i)Determine the order of the ARMA(p,q) process. [1]

(ii) Determine whether the process is stationary and/or invertible. [4]

(iii) Find the first 4 (ψ) weights and first 4 (π) weights of the process. [5]

(b) Show that the ACF of the ARMA (1,1) model

$$Z_t = \phi Z_{t-1} + a_t - \theta a_{t-1},$$

is given by

$$\rho_k = \frac{(1-\theta\phi)(\phi-\theta)}{1-2\theta\phi+\theta^2}\phi^{k-1}, \text{for } k \geq 1.$$

[10]

B9. Consider the following AR(2) model

$$Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + a_t,$$

where a_t is a sequence of independent and identically distributed random variables with mean 0 and variance σ_a^2.

(a) Given that $r_1 = 0.3571$ and $r_2 = 0.2214$, Use the method of moments to estimate ϕ_1 and ϕ_2. [10]

(b) Showing all your working, deduce that if $\phi_1 = \phi_2 = \frac{1}{12}$, then the autocorrelation function of Z_t is given by

$$\rho_k = \frac{45}{77}(\frac{1}{3})^{|k|} + \frac{32}{77}(-\frac{1}{4})^{|k|}, k = 0, \pm1, \pm2...$$

[10]

END OF QUESTION PAPER