NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SORS2103

FACULTY OF APPLIED SCIENCE

DEPARTMENT OF STATISTICS AND OPERATIONS RESEARCH

SORS2103 PROBABILITY THEORY

BSc. Operations Research & Statistics Part II

August 2024 SPECIAL SUPPLEMENTARY EXAMINATION

Time : 3 hours

Total Marks: 100

Candidates may attempt **ALL** Questions in Section **A** and at most **THREE** Questions in Section **B**. **For all questions where necessary clearly show your work to indicate how you obtained the answer**. You may use a calculator and Statistical Tables will be provided.

SECTION A: Attempt all questions in this section (40 marks).

A1. Suppose two events A and B, are such that $P(A) = 0.52$, $P(B) = 0.34$, and $P(A \cap B) = 0.18$.

(a) Find the probability that none of the two events occur. [2]

(b) Find the probability that at least one of the two events occur. [2]

(c) Find the probability that B does not occur if A occurs. [2]

A2. An explosion at a construction site could have occurred as the result of static electricity, malfunctioning of equipment, carelessness, or sabotage. Interviews with construction engineers analysing the risks involved led to the estimates that such an explosion would occur with probability 0.25 as a result of static electricity, 0.20 as a result of malfunctioning of equipment, 0.40 as a result of carelessness, and 0.75 as a result of sabotage. It is also felt that the prior probabilities of the four causes of explosion are 0.20, 0.40, 0.25, and 0.15. Based on all this information, what is

(a) the most likely cause of the explosion; [5]

(b) the least likely cause of the explosion. [1]

A3. (a) A manager at a local restaurant is considering giving Milton a raise. The manager estimates that there is 80% chance that Milton will quit if he does not get a raise. Is this empirical, classical or subjective probability? [1]

(b) In an experiment to test iron strengths, three different ores, four different furnace temperatures, and two different cooling methods are to be considered. Altogether, how many experimental configurations are possible? [1]

(c) Twenty players compete in a tournament.

(i) In how many ways can rankings be assigned to the top five competitors? [2]

(ii) In how many ways can the best five competitors be chosen (without being in any order)? [2]

(d) In a primary election, there are four candidates for mayor, five candidates for city treasurer, and two candidates for local attorney. In how many ways can a person vote if he exercises his option of not voting for any or all of these offices? [2]

(e) An experiment has three outcomes a, b and c. If outcome a is twice as likely as outcome b, and outcome b is three times as likely as outcome c, what are the probability values of the three outcomes? [3]

(f) A market research organisation claims that, among 500 shoppers interviewed, 308 regularly buy Product X, 266 regularly buy product Y, 103 regularly buy both, and 59 buy neither on a regular basis. Using a Venn diagram and filling in the number of shoppers associated with the various regions, check whether the results of this study should be questioned. [3]

(g) Show that there is no value of k for which

$$f(x,y) = \begin{cases} ky(2y-x), & for\ x = 0,3; y = 0,1,2, \end{cases}$$

can serve as the joint probability distribution of two random variables. [5]

A4. Suppose that the profit that a certain contractor will make on any job, in thousands of dollars, is a random variable X with probability density function given by

$$f(x) = \begin{cases} \frac{4x-x^3}{4}, & 0 < x < 2, \\ 0, & \text{otherwise.} \end{cases}$$

(a) On the average, how much profit can this contractor be expected to make per job? [3]

(b) Find the variance of X. [4]

(c) Should you be surprised if he makes a profit of less than \$400.00 on his next job? Why or why not? [2]

SECTION B: Attempt any three questions in this section (60 marks).

B5. (a) Suppose that A and B are independent events in an experiment with $P(A) = 0.4$ and $P(B) = 0.6$. Find$P(A \cup B)$. [2]

(b) Given the following cumulative distribution function
$F(x) = 1 - e^{-x} - xe^{-x}; 0 < x < \infty$,
find the probability density function of random variable X. [3]

(c) Let a random variable X have a probability density function

$$f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b, \\ 0, & \text{otherwise.} \end{cases}$$

(i) Show that the probability that it will take on a value more than $b - p(b - a)$ is equal to p. [3]

(ii) Find the moment generating function of the random variable X. [3]

(d) Let X and Y have a joint probability mass function

$$f(x,y) = \begin{cases} \frac{1}{42}(2x + y), & x = 0, 1, 2;\ y = 0, 1, 2, 3, \\ 0, & \text{otherwise.} \end{cases}$$

(i) Find the marginal distributions of X and Y. [3,3]

(ii) Find the conditional cumulative distribution function of X given $Y = 0$. [3]

B6. (a) Let the probability density function (pdf) of X be given by

$$f(x) = \begin{cases} \frac{kx^3}{(1+2x)^6}, & x > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

where k is an appropriate constant.

(i) Find the probability density of the random variable $Y = \frac{2X}{1+2X}$. [6]

(ii) Identify the distribution of Y, and thus determine the value of k. [2,2]

(b) Let X, Y be two continous random variables having the joint probability density function

$$\begin{cases} f(x,y) = 24xy, & 0 < x < 1,\ 0 < y < 1,\ x + y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Find the joint probability density of $Z = X + Y$ and $W = X$. [10]

B7. (a) If $var(X_1) = 5$, $var(X_2) = 4$, $var(X_3) = 7$, $cov(X_1, X_2) = 3$, $cov(X_1, X_3) = -2$, and X_2 and X_3 are independent, find the covariance of $Y_1 = X_1 - 2X_2 + 3X_3$ and $Y_2 = -2X_1 + 3X_2 + 4X_3$. [3]

(b) Many veterans who bravely served in Iraq during Operation Enduring Freedom or Operation Iraqi Freedom have developed symptoms of post-traumatic stress disorder (PTSD) or similar ailments. A large study of returning veterans showed that 52.7% of soldiers suffered from a PTSD or similar ailment after serving Iraqi. A random sample of 200 veterans from Iraq were selected for additional research. The number suffering from PTSD or a similar condition was recorded. (Source: Frayne, et al. (2010). Medical care needs of returning veterans with PTSD: their other burden. Journal of General internal Medicine. 26(1).)

(i) Explain why this is a binomial experiment. [4]

(ii) What is the mean of X, the number of people suffering from PTSD in a sample of 200 veterans? [2]

(iii) Interpret the mean. [1]

(iv) Compute the standard deviation of X. [1]

(v) Would it be unusual to find 110 individuals suffering from PTSD in a group of 200 veterans? [2]

(c) The number of people, X, entering the intensive care unit at a particular hospital on any single day possesses a Poisson distribution with mean equal to four persons per day.

(i) What is the probability that the number of people entering the intensive care unit on a particular day is equal to 2? [3]

(ii) Is it likely that X will exceed 10? Explain. [2]

(iii) What is the probability that the number of people entering the intensive care unit on a particular half-day is equal to 2? [2]

B8. (a) If we let $k\sigma = c$ in Chebyshev's theorem, what does this theorem assert about the probability that a random variable will take on a value between between $\mu - c$ and $\mu + c$? [3]

(b) Given the moment generating function $M_X(t) = e^{3t+8t^2}$, find the moment generating function of the random variable $Z = \frac{1}{4}(X - 3)$. [3]

(c) If the probability is 0.20 that a certain bank will refuse loan application, use the normal approximation to determine (to 3 decimal places) the probability that the bank will refuse at most 40 of 225 loan applications. [4]

(d) The probability density function of a continous random variable, X, is given by

$$f(x) = \begin{cases} kx^2, & 0 \leq x < 1, \\ k, & 1 \leq x < 2, \\ k(3-x) & 2 \leq x < 3, \\ 0, & \text{otherwise.} \end{cases}$$

(i) Show that $k = \frac{6}{11}$. [4]

(ii) Find the distribution function (cdf) of the random variable X. [6]

END OF QUESTION PAPER