NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCE

DEPARTMENT OF STATISTICS AND OPERATIONS RESEARCH

SORS 2104 : OPERATIONS RESEARCH TECHNIQUES

BSc OPERATIONS RESEARCH & STATISTICS: PART II

December 2024 EXAMINATION

Time : 3 hours

Total Marks: 100

Candidates may attempt **ALL** Questions in Section **A** and at most **THREE** Questions in Section **B**. Show all working where necessary. You may use a calculator, Graph paper and Statistical Tables will be provided.

SECTION A: Attempt all questions in this section (40 marks).

A1. (i) Define the term slack of an activity and critical path as used in project management. [4]

(ii) Explain the terms latest start time and earliest finish time of a project. [4]

(iii) The expected time to complete a project is 21 weeks with a standard deviation of 2.15 weeks. What is the probability that the project will be delayed? [5]

A2. Construct a network diagram for the network connections in Table1. [3]

(a) Find the minimum spanning tree. [3]

(b) The shortest route from E to all the other nodes. [5]

Table 1: Network connections

	A	B	C	D	E
A	-	6	9	4	10
B	6	-	9	8	7
C	9	9	-	6	12
D	4	8	6	-	3
E	10	7	12	3	-

A3. (a) What is the difference between PERT and CPM [2]

(b) State and explain any 3 rules for drawing a network diagram. [6]

A4. A project consists of the following tasks

Table 2: Precedence table for a roofing project

Task	Duration(days)	Predecessors
A	3	None
B	5	A
C	2	A
D	4	B,C
E	6	C
F	3	D,E

(a) Draw a Gantt chart for this project.Assuming work begins on day 1. [5]

(b) Determine the earliest day each task can start and finish. [5]

SECTION B: Attempt any three questions in this section (60 marks).

B5. Arrivals at a fuel service station are considered to be Poisson at an average time of 4 min between arrivals. The length of service is distributed with a mean of 2 min

    (a) Determine the:

    (i) expected fraction of the day that the fuel attendant will be serving.   [3]

    (ii) expected number of vehicles in the system.   [4]

    (iii) expected waiting time in the system.   [3]

    (b) What is the probability that:

    (i) a vehicle will have to wait in the queue?   [3]

    (ii) there are more than 3 vehicles in the system?   [3]

    (iii) a vehicle will have to wait more than 3min in the queue before service?   [4]

B6.   (a) State and explain 3 effects of understocking in business.   [6]

    (b) Initiating an order for cleaning detergents costs$25. Detergent costs $1 per liter,$0.55 holding cost per year per liter. Demand 260 000litres per year.

    (i) What is the most economical amount of detergent to order?   [4]

    (ii) How many orders should be placed in a year?   [3]

    (iii) How often should an order be placed?   [3]

    (iv) Calculate the total annual costs.   [4]

B7. (a) What is the difference between optimistic time and pessimistic time in reference to project activity times? [4]

(b) Using the information in Table 3

Table 3: Precedence table for a building project

Task	Duration(days)	Predecessors
A	5	-
B	15	A
C	25	B
D	15	B
E	30	B
F	10	C,D
G	10	E,F
H	5	G
I	5	H

(i) Draw the network diagram for the project. [6]

(ii) Determine the critical path and the planned duration of the project in weeks. [6]

(iii) Identify any non critical tasks and the float on each. [4]

B8. (a) Explain the term 'optimal decision' as used in decision analysis. [2]

(b) What is a pay off table? [2]

(c) A decision maker is faced with four decision alternatives and four states of nature and develops the following profit pay off table.

Table 4: Profit pay off table

Decision alternative	State of nature			
	s_1	s_2	s_3	s_4
d_1	140	95	100	50
d_2	110	100	80	70
d_3	90	100	105	101
d_4	80	105	110	130

If the decision maker knows nothing about the probabilities of the four states, what is the recommended decision using:

(i) optimistic approach. [2]

(ii) minimax approach. [6]

(iii) Suppose the decision maker obtains information that $P(s_1) = 0.3$, $P(s_2) = 0.2$, $P(s_3) = 0.1$, $P(s_4 = 0.4$. Use the expected value approach to determine the optimal decision. [8]

LIST OF SELECTED FORMULAE

(i) $L_q =$

$$\frac{\lambda^2}{\mu(\mu - \lambda)}$$

(ii) $W_q =$

$$\frac{L_q}{\lambda}$$

(iii) $L = L_q +$

$$\frac{\lambda}{\mu}$$

(iv) $W = W_q +$

$$\frac{1}{\mu}$$

(v) $D =$

$$\frac{\mu}{(\mu - \lambda)}$$

(vi) $P_0 = 1 -$

$$\frac{\lambda}{\mu}$$

(vii) $P_n = P_0 -$

$$(\frac{\lambda}{\mu})^n$$

(viii) $P_{normore} = P_0$

$$(\frac{\lambda}{\mu})^n$$

(ix) $Q^* = \sqrt{\frac{2DC_0}{C_h}}$