NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SORS 4101

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF STATISTICS AND OPERATIONS RESEARCH

SORS 4101: DECISION ANALYSIS

BSc. OPERATIONS RESEARCH: PART IV

DECEMBER 2024 EXAMINATION

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B. Each question should start on a fresh page

---

SECTION A: Answer all questions in this section (40 marks).

A1. Consider the following payoff matrix,

$$\begin{bmatrix} 15 & 10 & 0 & -6 & 17 \\ 3 & 14 & 8 & 9 & 2 \\ 1 & 5 & 14 & 20 & -3 \\ 7 & 19 & 10 & 2 & 0 \end{bmatrix}$$

Find the solutions obtained by each of the following criteria to find the optimal decision

(a) The Laplace criterion. [3]

(b) The Savage Regret criterion. [4]

(c) The Hurwicz criterion. [3]

A2. Suppose we are given the following table of utilities from a certain investor in a certain volatile economic environment characterised by four positions of the service demand offered. What would be the attitude of such a decision maker towards risk?

Table 1: Table of Utilities

Investment Portfolios	Economic Volatility			
	V.Low	Low	High	V.High
No Investment	6.3	6.3	6.3	6.3
1	5.0	7.9	7.9	7.9
2	3.0	6.9	9.0	9.0
3	0.0	5.7	8.3	1.0

Where U(-30)=0, U(-20)=3.0, U(-10)=5.0, U(-5)=5.7, U(0)=6.3, U(5)=6.9, U(20)=8.3, U(15)=0, U(30)=0 and U(45)=10. [5]

A3. (a) Explain how a bargaining process can be modeled as a game theory problem. [4]

(b) Determine the saddle point of the following game problem.

$$\begin{pmatrix} 6 & 5 & 9 \\ -3 & 1 & 12 \end{pmatrix}$$

[3]

(c) Who is the winner in this game? [1]

A4. Find the range of values of p and q which will render the entry (2,2) a saddle point for the game;

$$\begin{bmatrix} 2 & 4 & 5 \\ 10 & 7 & q \\ 4 & p & 6 \end{bmatrix}$$

[5]

A5. Solve, using the concept of dominance, the game whose payoff matrix is given below: [12]

$$\begin{bmatrix} 3 & 2 & 0 & 0 \\ 3 & 4 & 2 & 4 \\ 4 & 2 & 1 & 6 \\ 0 & 4 & 1 & 8 \end{bmatrix}$$

SECTION B: Answer any THREE questions in this section (60 marks).

B6. A firm has three investment alternatives. The payoff table (in $1000's) and associated probabilities are as follows.

Table 2: Payoff Table and Associated Probabilities

Investments	Economic Conditions		
	Up	Stable	Down
d_1	100	25	0
d_2	75	50	25
d_3	50	50	50
Probabilities	0.40	0.30	0.30

For the lottery having a payoff of $100,000 with probabilit p and $0 with probability $(1-p)$, two decision makers expressed the following indifference probabilities.

Table 3: Indifference Probabilities

Profit ($1000's)	Indifference Probability	
	Decision Maker A	Decision Maker A
75	0.80	060
50	0.60	0.30
25	0.30	0.15

(a) Find the most preferred decision for each decision maker using the expected utility approach. [9]

(b) Why don't decision makers A and B select the same decision alternative? [1]

(c) An upcoming horticulturist finds from past data that the cost of producing a case of butternut is R25 and can sell them at R30 each if it is sold within the week the butternuts are harvested. Failing to sell them the week they are harvested means that they have got to be disposed at R20 per case. The same data also indicated the following weekly sales.

Table 4: Weekly Sales

Weekly Sales	≤ 3	4	5	6	7	≥ 8
No. of Weeks	0	10	20	40	30	0

Find the optimum number of cases the horticulturist should harvest each week. [10]

Table 5: Profit from each crop

Rainfall	Maize	Tomatoes	Sugarcane
Substantial	7	2.5	4
Moderate	3.5	3.5	4
Light	1	4	3

B7. A farmer is attempting to decide which of three crops (Maize, Tomatoes and Sugarcane) she should plant on her 100 acre farm. The profit from each crop is strongly dependent on the rainfall during the growing season. She has categorised the amount of the rainfall as substantial, moderate or light. She estimates the profit (USD'000) from each crop as shown in the table below.

Based on the weather in previous seasons and the current projection for the coming season, she estimates the chances of substantial rainfall as 20%, that of moderate as 30% and that of light rainfall as 50%. Furthermore, services of the meteorological department could be employed to provide a detailed survey of current rainfall prospects i.e. P(Crop/Rainfall) and the forecasts are as below.

Table 6: Type of Rainfall

Rainfall	Maize	Tomatoes	Sugarcane
Substantial	0.70	0.25	0.05
Moderate	0.30	0.60	0.10
Light	0.10	0.20	0.70

(a) Determine the optimal decision as to which crop to plant. [6]

(b) Determine whether it would be economical or not for the farmer to hire the services of the state meteorological department. [12]

(c) Find the efficiency of the information provided by the meteorological department. [2]

B8. (a) Use the graphical method in solving the following game. Which player wins the game

$$\begin{bmatrix} 2 & 2 & 3 & -2 \\ 4 & 5 & 2 & 6 \end{bmatrix}$$

[8]

(b) Find the optimal strategies of two players involved in the following game using the simplex method.

$$\begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$$

Hence or otherwise find the value of the game. [12]

B9. (a) A Statistician is observing values from a Bin(2, p) distribution. He knows that p is equal either to 1/4 or to 1/2, and he is trying to choose between these two values. He observes a single value x from the distribution. He proposes to use one of the following four decision functions.

$$d_1(x) \quad : \qquad \text{set } p = 1/4 \text{ when } x = 0$$
$$: \qquad \text{set } p = 1/2 \text{ when } x = 1 \text{ or } 2$$

$$d_2(x) \quad : \qquad \text{set } p = 1/4 \text{ when } x = 0 \text{ or } 1$$
$$: \qquad \text{set } p = 1/2 \text{ when } x = 2$$

$$d_3(x) \quad : \quad \text{set } p = 1/4 \text{ when } x = 0, \ 1, \text{ or } 2$$

$$d_4(x) \quad : \quad \text{set } p = 1/2 \text{ when } x = 0, \ 1, \text{ or } 2$$

If he incorrectly concludes that p=1/4, he suffers a loss of 1. If he incorrectly concludes that p=1/2, he suffers a loss of 2. Find the risk function for each decision function, and find the decision function that minimises the maximum expected loss. [10]

(b) Two Fast food competitors, Macs and Thelwalls are competing for an increased market share. The payoff matrix. shown in the following table, shows the increase in the market share for Macs and decrease in market share for Thelwalls.

Table 7: Macs and Thelwalls Payoff Matrix

Macs	Thelwalls			
	Give Coupons	Decrease Price	Maintain Current Strategy	Increase Advertising
Give Coupons	2	-2	4	1
Decrease Price	6	1	12	3
Maintain Present Strategy	-3	2	0	6
Increase Advertising	2	-3	7	1

Use the algebraic method to find the optimal strategies for both manufactures and the manufacturer that gets the bigger market share.. [10]

END OF QUESTION PAPER