NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF STATISTICS AND OPERATIONS RESEARCH

SORS4102: STATISTICAL INFERENCE

BSc. OPERATIONS RESEARCH and STATISTICS: Part IV

AUGUST 2024 SPECIAL EXAMINATION

Time: 3 Hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B. Each question should start on a fresh page.

SECTION A [40 marks]

1. Define following terms as they are used in statistical inference:

(a) Type I error, [2]

(b) Estimator, [3]

(c) efficient statistic, and a [2]

(d) parameter. [3]

2. (a) Explain the significance of expressing a distribution as a member of the exponential family. [5]

(b) Show that the Poisson distribution is a member of the exponential family. [10]

[HINT: $f(x,p)=\frac{\lambda^x e^{\lambda}}{x!}$]

3. (a). What are advantages of maximum likelihood method of estimation (MLE) in finding estimators for population parameters? [5]

(b). Let $X_1,...,X_n$ be a random sample from some density distribution which has mean μ and variance σ^2.

Show that $\sum_{i=1}^{n} a_i X_i$ is an unbiased estimator of μ for any set of known constants $a_1,...,a_n$ satisfying $\sum_{i=1}^{n} a_{ii} = 1$. [10]

SECTION B

Attempt any **THREE** questions from this section **[60 marks]**

4. Let X be a single observation from the density function:

$f(x,\theta) = \theta x^{\theta-1}$, $\theta > 0$.

(a) Find the pivotal quantity. [10]

(b) Use the answer in (a) to find a confidence interval estimator for θ. [5]

(c) Show that $(\frac{Y}{2}, Y)$ is a confidence interval for θ. [5]

5. Suppose $X_1,...,X_7$ is a random sample obtained from a density function:

$$f(x,\theta) = \begin{cases} (\theta+1)x^{\theta}, & if \quad 0 \le x \le 1, \\ 0, & otherwise. \end{cases}$$

To test:

$H_0 : \theta = 2$ versus $H_1 : \theta > 2$.

Using the decision γ : reject H_0 if $X \ge 4$.

(a) Find α for this test. [10]

(b) What proportion of the time would such a decision rule lead to a Type I error? [10]

6. Let $X_1,...,X_n$ be a random sample obtained from a Poisson distribution with probability with parameter λ. Given that:

$\theta_1 = \frac{X_1 + 2X_2 + 3X_3}{6}$ and $\theta_2 = \frac{X_1 + X_2 + X_3}{3}$.

(a) Show that both θ_1 and θ_2 are unbiased estimators of θ. [10]

(b) Find the efficiency of θ_1 relative to θ_2. [10]

7. Suppose $X_1,...,X_{10}$ be independent and identically distributed observations from a distribution with probability density function:

$$f(x,\theta) = \frac{\theta}{5X^4} e^{-\frac{\theta}{x}}, \quad 0< x <\infty, \ \theta > 0.$$

(i) Find the maximum likelihood estimator (MLE) for θ. [8]

(ii) Find the Fisher information for θ. [7]

(iii) State the Cramer-Rao lower bound. Use a few sentences to explain the role of the lower bound. [5]

END OF QUESTION PAPER