NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SORS 4205

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF STATISTICS AND OPERATIONS RESEARCH

SORS 4205: NON-LINEAR PROGRAMMING

BSc. Operations Research: Part IV

MARCH 2025 EXAMINATION

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B. Each question should start on a fresh page

SECTION A: Answer all questions in this section (40 marks).

A1. State any Characteristics of Non-Linear Functions you know. [4]

A2. Determine and classify the extreme point of the following function:

$$f(X) = 2x_1^2 - x_2^2 + 3x_3^2 + 5x_1x_2 - 4x_1x_3 + 6x_2x_3$$

[5]

A3. Use the golden search to solve the following problem.

Max $f(X) = -x^2 - 1$ in the interval $-1 \leq x \leq 0.75$

with the final interval of uncertainty having a length less than 0.25 [10]

A4. Use the Lagrange method to solve the following problem.

$$\text{Maximize } Z = 4x_1 - x_1^2 + 8x_2 - x_2^2$$

subject to

$$x_1 + x_2 = 2$$

$$x_1, x_2 \geq 0$$

[5]

A5. Consider the following Non-Linear Programming problem.

$$\text{Maximize } f(Y) = \ln(y_1 + 1) + y_2$$

subject to

$$2y_1 + y_2 \leq 3$$

$$y_1 \geq 0$$

$$y_2 \geq 0$$

(a) Write down the Karush-Kuhn-Tucker (KKT) necessary and sufficient conditions for the above problem. [10]

(b) Hence find the Optimal solution. [6]

SECTION B: Answer any THREE questions in this section (60 marks).

B6. Use the Dichotomous method to maximize the following unimodal function,

$$\mathrm{f(X)} = \mathrm{xsin}\ \pi x, 1.5 \leq x \leq 2.5$$

with $(\Delta = 0.5)$. [20]

B7. Consider the following quadratic function:

$$\text{Maximize } \mathrm{Z} = 2\mathrm{x}_1 + x_2 - x_1^2$$

s.t

$$2\mathrm{x}_1 + 3x_2 \leq 6$$

$$2\mathrm{x}_1 + x_2 \leq 4$$

$$\mathrm{x}_1, x_2, x_3 \geq 0$$

(a) Use the necessary conditions for Quadratic Programming to formulate the given function. [5]

(b) Use the Wolfe's method to solve the quadratic problem. [15]

B8. (a) Show how the following quadratic function can be made seperable:

$$\mathrm{f(T)} = \mathrm{t}_1^2 + t_2^2 - t_1 t_2$$

[2]

(b) Use the Geometric programming method to find the optimal solution to:

$$\text{Minimize f(X)} = 7\mathrm{x}_1 x_2^{-1} + 3x_2 x_3^{-2} + 5x_1^{-3} x_2 x_3 + x_1 x_2 x_3$$

[8]

(c) Solve the following problem by linear combinations method

$$\text{Max f(X)} = \text{x}_1^3 + x_2^2 - 3x_1x_2$$

subject to

$$3\text{x}_1 + x_2 \leq 3$$

$$5\text{x}_1 - 3x_2 \leq 5$$

$$\text{x}_1, x_2, x_3 \geq 0$$

[10]

Let $\text{X}^{(0)} = (1, 1)$. *Perform two iterations.*

B9. A company wants to allocate its advertising budget between two platforms: Social Media (A) and Television (B) to maximize customer engagement while staying within budget constraints.

The relationship between advertising spend and customer engagement follows a diminishing returns model, meaning that each additional dollar spent generates less engagement than the previous dollar. The engagement functions (measured in thousands of new customers reached) are given as:

$$\text{E}_A(x_1) = 20\sqrt{x_1}, \quad E_B(x_2) = 30\sqrt{x_2}$$

where:
$\text{x}_1 =$ *dollars spent on Social Media*
$x_2 =$ *dollars spent on Television*

The total advertising budget is \$1 000 000, meaning:

$$\text{x}_1 + x_2 \leq 1,000,000$$

The company wants to maximize customer engagement while ensuring a minimum of \$200,000 is allocated to each platform.

(a) Formulate this as a non-linear programming problem. [8]

(b) Use a suitable algorithm to determine how much is to be allocated to each advertising platform in order to maximize customer engagement. [12]

END OF QUESTION PAPER