# NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SORS 5102

FACULTY OF APPLIED SCIENCES

**DEPARTMENT OF STATISTICS AND OPERATIONS RESEARCH**

SORS 5102: STOCHASTIC MODELLING

MSc. Operations Research and Statistics: Part I

NOVEMBER 2024 EXAMINATION

Time : 3 hours

Answer **ANY FOUR** questions .
Answers for each question should start on a fresh page.

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**A1.** (a) Given that $X_n = \sum_{k=1}^{n} Z_k$, where $Z_k$ is the independent outcome of the $k^{th}$ roll of a fair die, calculate $\mathrm{P}(X_1 = 2, X_2 = 4, X_3 = 6)$. [5]

(b) A cow can graze in any of the three paddocks 1, 2 and 3 at a farm. If the cow grazes at paddock 1, it will graze at paddock 2 on the next day with a probability of one. Similarly, if the cow grazes at paddock 3, it will graze at paddock 2 on the following day with a probability of one. However, if the cow grazes at paddock 2, there is a 25% chance that it will graze at paddock 1 on the next day otherwise it will graze at paddock 3.

i) Draw the transition diagram of the grazing process. [5]

ii) Given that $X_n$ denotes the position of the cow on the $n^{th}$ day of grazing and assuming that $X_0 = 1$, calculate $\mathrm{P}(X_0 = 1, X_1 = 2, X_2 = 3)$. [5]

iii) What is the probability of being in paddock 3 on the long run. [10]

**A2.** (a) The arrival times of cars at a Police Road Block follows a Poisson process with a mean arrival rate of 3 cars per hour.

i) What is the probability that no car will arrive at the road block in the next hour? [2]

ii) What is the probability that 12 or less cars will arrive at the road block in the next five hours? [10]

(b) The arrival of students at NUST is modelled by a non-homogeneous Poisson process with an intensity function :-

$$\lambda(t) = 3 + sin(\frac{t}{4})$$

.

Given that t=0 is 08:00 am,

i) What is the probability that no student will arrive in the next 1 hour? [3]

ii) What is the probability that 12 or less students will arrive in the next five hours? [10]

**A3.** (a) Consider a gambler who at each play of the game has probability p of winning one unit and probability q = 1-p of losing one unit. Assuming that successive plays of the game are independent, what is the probability that, starting with i units, the gambler's fortune will reach N before reaching 0? [15]

Hint: Derive the formula for a Gambler Ruin Problem.

(b) Samukeliso and Bathabile decide to flip ZiG coins. The one coming closest to the wall wins. Samukeliso, being the better player, has a probability 0.6 of winning on each flip.

i) If Samukeliso starts with five ZiG coins and Bathabile with ten ZiG coins, what is the probability that Bathabile will be ruined? [4]

ii) If Samukeliso starts with ten ZiG coins and Bathabile with twenty ZiG coins, what is the probability that Bathabile will not be ruined? [6]

**A4.** (a) Suppose $X_n$, n=0,1,2...... is a homogenous Markov Chain, prove that $P_{i.j}^{m+n} = \sum P_{i,j}^{m} P_{k.j}^{n}$. [10]

(b) The internally displaced persons migration process in a war torn country with four provinces was classified to be an absorbing markov chain governed by the following transition matrix:-

$$P = \begin{pmatrix} 0.4 & 0 & 0.3 & 0 \\ 0.3 & 1 & 0.4 & 0 \\ 0.1 & 0 & 0.2 & 0 \\ 0.2 & 0 & 0.1 & 1 \end{pmatrix}.$$

. [15]

Calculate the stable transition matrix for the chain.

**A5.** (a) Given that a population of individuals possesses two genes either type A or type a. In their outward appearance, type A is dominant and type a is recessive. An individual will have only the outward characteristics of the recessive gene if its pair is aa. Suppose that the population has stabilized and that the percentages of individuals having respective gene pairs AA, aa, and Aa are p, q, and r. Let $S_{11}$ denote the probability that an offspring of two dominant parents will be recessive and let $S_{10}$ denote the probability that the offspring of one dominant and one recessive parent will be recessive. Show that $S_{11} = S_{10}^2$. [12]

(b) A university has N employees where N is a large number. Each employee has one of three possible job classifications (Technician, Administrator and Lecturer). The employee job classifications change independently according to a Markov chain with transition probabilities

$$P = \begin{pmatrix} 0.7 & 0.2 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.4 & 0.5 \end{pmatrix}.$$

What percentage of employees will be in each job classification on the long run? [13]

END OF QUESTION PAPER