NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF STATISTICS AND OPERATIONS RESEARCH

SORS5103: INDUSTRIAL STATISTICS

MSc. OPERATIONS RESEARCH and STATISTICS

DECEMBER 2024

Time: 3 Hours

Candidates should attempt **ANY FIVE** questions. All questions carry equal marks. Each question should start on a fresh page.
TOTAL MARKS: 100

Q1. Sixteen patients with advanced stomach carcinoma were randomized to receive one of two chemotherapies (Group A or Group B). The survival times from treatment (in weeks) are (+ denotes a censored observation):

Group A: 63+, 59+, 57+, 40, 37, 33, 21+, 11
Group B: 57+, 51+, 44+, 32, 27, 27+, 10+, 6

(a) Two reasons why an observation might be censored include:
(i) administrative: study closes before patient dies;
(ii) loss to follow-up during study due to patient leaving the area.
State for each whether the assumption of statistical independence with survival time is plausible or not and the basis for your statement. [10]

(b) Construct (arithmetically) and plot (very roughly) the Kaplan-Meier survival curve for Group B. [10]

Q2. (a) One of the themes of Industrial Statistics is an understanding of effect modification. A term for this in analysis of variance parlance is "interaction". A two-way factorial design with replicate observations for each combination of factor I and II permits the discovery of "interaction". We can get a visual sense of this by constructing a plot of the group means. Consider a two-way factorial design with factor I at two levels designated "A_1" and "A_2" and factor II at two levels, designated "B_1" and "B_2". For each of the following three scenarios of main effects and interaction, shown in Figure 1, provide the following:

(i) Identify the correct graphical summarisation of the means, [5]

(ii) Write down the correct model using the notation μ, α_i, β_j, $(\alpha\beta)_{ij}$ and σ_e^2 as appropriate. [5]

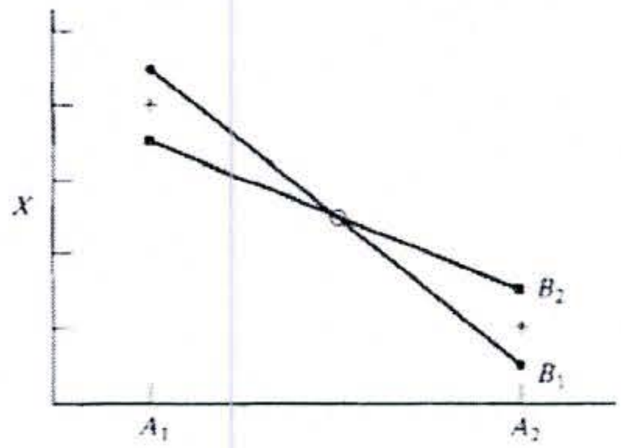
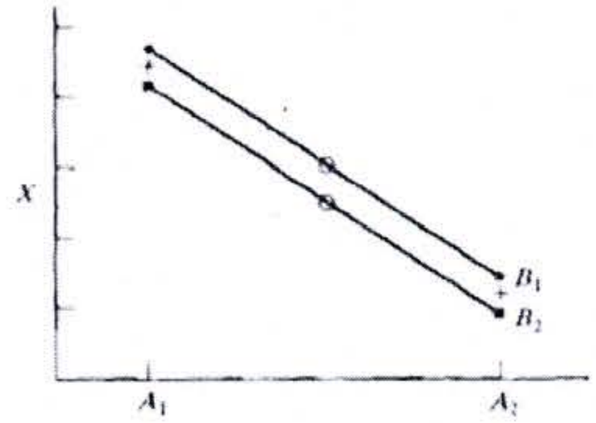
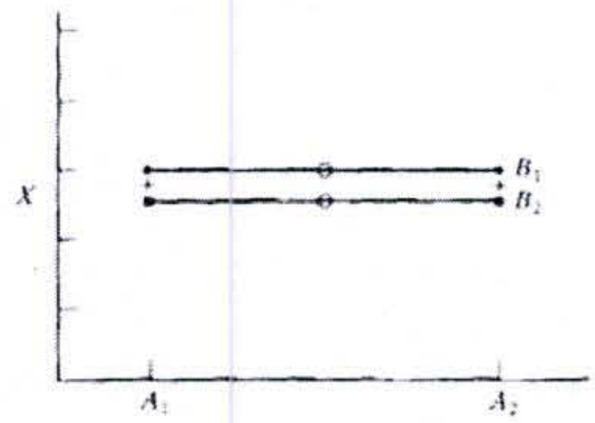
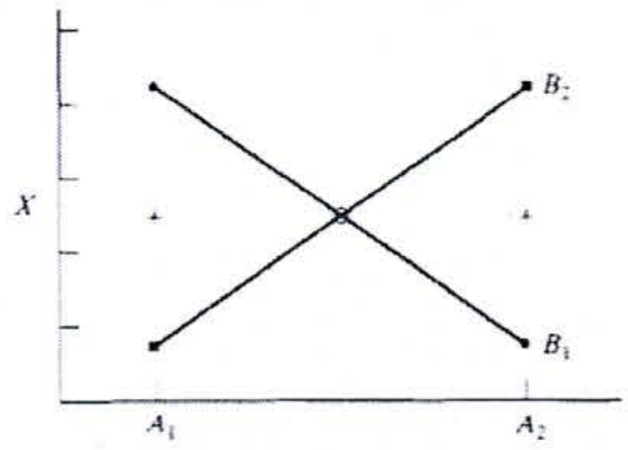
Plot 1	Plot 2
	
Plot 3	**Plot 4**
	

Figure 1: Effect modification

[**EXAMPLE: Plot 3:** (i) shows no effect of factor 1, small effect of factor II and there is no interaction, (ii) $Y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ijk}$]

(b) Explain **Yates** technique for calculating sum of squares due to main effects and interaction effects. Also write down the ANOVA table in case of a 2^3 factorial experiment with r replications. [10]

Q3 (a) Discuss how the efficiency of an experiment can be increased by using local control and by increasing the number of replications. Determine its efficiency of LSD relative to RBD taking columns as blocks and CRD. [10]

(b) An Industrial Engineer has studied the effect of speed (B), feed (C) and Depth of Cut (A) on the surface finish of a machined component using a three-factor factorial design. All the three factors were studied at two levels each. The surface roughness measurements (microns) from two replications are given in Table 1. Analyse the data and draw conclusions, use $\alpha = 0.05$. [10]

Table 1: Effect of speed, feed and depth on surface finish

		Speed (B)			
		100		120	
		Feed (C)		Feed (C)	
		0.20	0.25	0.20	0.25
Depth (A)	0.15	54	41	59	43
		52	58	61	55
	0.20	86	62	82	65
		82	64	75	77

Q4 (a) A Shewhart chart has center-line at 10 with UCL = 16 and LCL = 4. Suppose you wish to supplement this chart with an EWMA control chart using $\lambda = 0.1$ and the same control limit width in σ-units as employed on the chart. What are the values of the steady-state upper and lower control limits on the EWMA chart? [10]

(b) Consider an EWMA control chart. The target value for the process is $\mu_0 = 51$ and $\sigma = 2$.

Use Table 2 to answer the following questions:

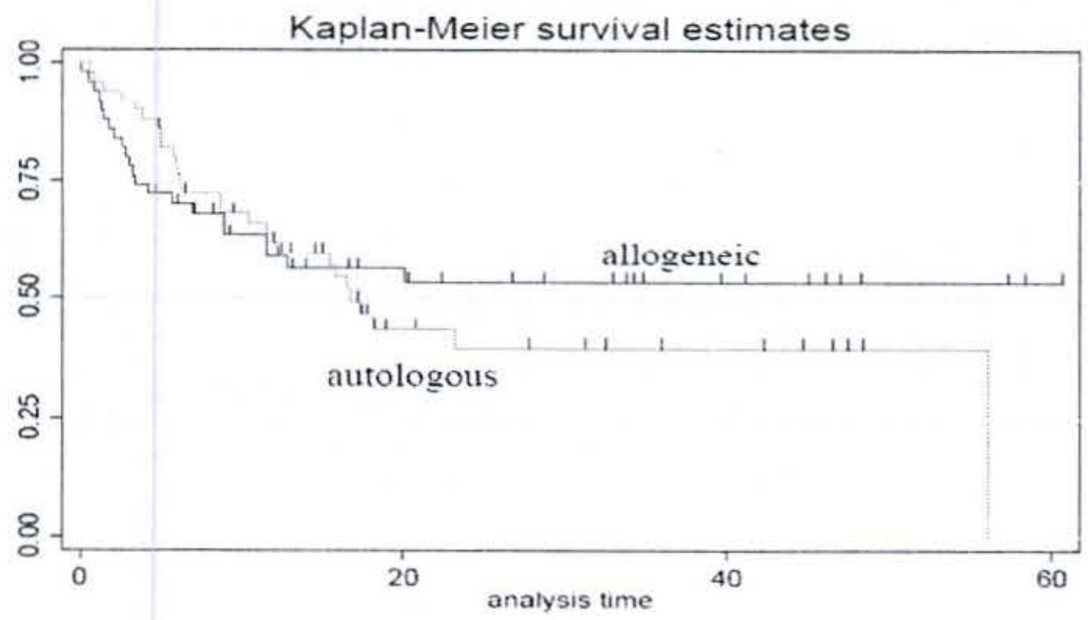
(i) If the sample is n=1. Would you prefer an EWMA chart with $\lambda = 0.1$ and L= 2.81 or $\lambda = 0.5$ and L= 3.07 to detect a shift in the process mean to $\mu = 53$. [5]

(ii) If the sample size increases to n=9, which chart in part (a) would you prefer? [5]

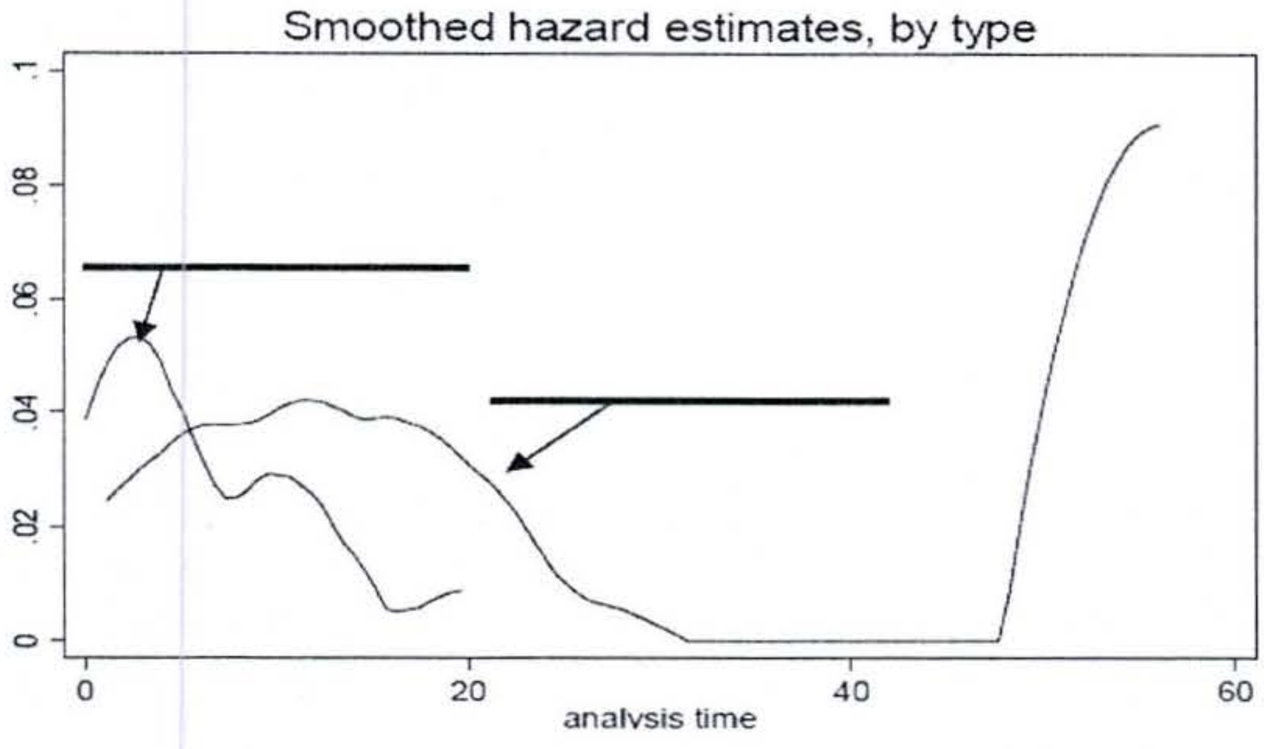
Table 2: ARL for an EWMA control chart

Shift in mean (multiple of $\sigma_{\bar{x}}$)	**λ =0.5 L=3.07**	**λ =0.1 L=2.81**
0	500	500
0.25	255	106
0.5	88.8	31.3
0.75	35.9	15.9
1	17.5	10.3
1.5	6.53	6.09
2	3.63	4.36
3	1.93	2.87

Q5. A group of 101 leukemia patients who received allogeneic vs autologous (type) transplants were followed up from transplant until death or relapse (Klein & Moeschberger, 1997). Time was recorded in months. The figure below shows the Kaplan-Meier estimates of the corresponding survival functions:



(a) Label the hazards as "allogeneic" or "autologous" in the plot below. Justify your answers [10]



(b) The failure of a unit in a system means the termination of the unit's ability to perform the required function. Suppose that the time to failure T has the probability density function *f(t)*. The failure distribution function is the probability of an item failing in the time interval $0 \leq \tau \leq t$:
Show that the survival function written as a function of the hazard function is:

$$S(t) = e^{-\int h(t)dt}.$$ [10]

Q6. Suppose that your assistant has been maintaining three C- control charts: one to monitor the number of defects of type A per product (C_A), one to monitor the number of defects of type B per product (C_B), and the third chart to monitor the number of defects of type C per product (C_c). Let us assume that a type A defect costs your company $\$a_1$ to fix, a type B defect costs $\$a_2$ to fix, and a type *C* defect costs $\$a_3$. Suppose that you want to maintain one control chart to monitor the total amount in dollars required to fix all the defects of types A, B, and C per product. That is, the test statistic to be plotted on the new control chart is total cost per product = $a_1 C_A + a_2 C_B + a_3 C_c$, where C_A, C_B and C_c are the actual number of defects of types A, B, and C per product, respectively. Assume that the mean number of defects of types A, B, and C per product are C_{0A}, C_{0B}, and C_{0C}, respectively.

(a). What are the limits of the new control chart? Assume that $\alpha = 0.005$. Assume also that C_A, C_B, and C_C are independent. [10]

(b). If $a_1 = \$5.00$, $a_2 = \$15.00$, and $a_3 = \$10.00$; the mean number of defects of type *A* per product (C_{0A}) = 3; the mean number of defects of type B per product (C_{0B}) = 4; and the mean number of defects of type C per product (C_{0C}) = 8, what are the control limits of this new chart? (α =0.005.) [10]

END OF QUESTION PAPER