MSc. Operations Research and statistics: Part II
NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY
SORS 6103

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF STATISTICS AND OPERATIONS RESEARCH

SORS 6103: FINANCIAL MODELLING

DECEMBER 2024 EXAMINATION

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B. Each question should start on a fresh page.

SECTION A

Answer ALL questions in this section [40 Marks]

A1. Define the following terms:

   (a) Arbitrageur,   [3]

   (b) call option,   [3]

   (c) forward contract.   [3]

A2. Suppose that USD/sterling spot and forward exchange rates are as follows:

Spot	1.5580
90-day forward	1.5556
180-day forward	1.5518

What opportunities are open to an arbitrageur in the following situations?

   (a) A 180-day European call option to buy £1 for $1.42 costs 2 cents.   [5]

   (b) A 90-day European put option to sell £1 for $1.49 costs 2 cents.   [5]

A3. (a) The Ljung-Box (LB) test checks for correlation in a series z_t. Describe how the LB test is calculated, using a general number of k lags, including it's asymptotic density. [3]

(b) When considering an ARMA(p, q)-GARCH(m,s) model, the LB test can be used on either

    i. r_t, the percentage return of the series;

    ii. r_t^2, the squared returns;

    iii. a_t, the residual after accounting for the ARMA(p, q) part;

    iv. a_t^2, the squared residuals;

    v. $\epsilon_t = a_t/\sigma_t$, the standardized residual, where σ_t is the standard deviation at time t of the GARCH(m,s) component, and

    vi ϵ_t^2, the squared standardized residual.

Discuss for each of these possible series what null-hypothesis the LB test is testing, and whether the signal of the LB test is a sensible one (a test is not sensible if there is a different series giving a clearer signal for the same null hypothesis). [5]

(c) The GARCH(m,s) is an extension of the ARCH(m) model. Discuss the extension, and explain why in practice this extension provided such a great improvement over the original ARCH(m) model. [3]

(d) Alternatively, the occurrence of ARCH(m) effects can be tested using a Lagrange Multiplier (LM) test. For this test, first an ARMA(p, q) model is estimated on the data, and for the residuals a_t a regression is run explaining a_t^2 from a constant and m lags $a_{t-1}^2, ..., a_{t-m}^2$. This test can also be used to detect GARCH(1, 1) effects. Discuss what choice of m might be optimal for detecting GARCH(1,1). (Hint: The answer is not $m = 1$). [3]

A4. What is arbitrage? Explain the arbitrage opportunity when the price of a dually listed mining company stock is $50 (USD) on the New York Stock Exchange and $52 (CAD) on the Toronto Stock Exchange. Assume that the exchange rate is such that $1 US dollar equals $1.01 Canadian dollar. Explain what is likely to happen to prices as traders take advantage of this opportunity. [7]

SECTION B

Answer any THREE questions in this section [60 Marks]

B5. Suppose that the price of a non-dividend-paying stock is $32, its volatility is 30%, and the risk-free rate for all maturities is 5% per annum. Use DerivaGem to calculate the cost of setting up the following positions:

(a) A bull spread using European call options with strike prices of $25 and $30 and a maturity of 6 months. [8]

(b) A bear spread using European put options with strike prices of $25 and $30 and a maturity of 6 months. [6]

(c) A butterfly spread using European call options with strike prices of 25,30, and $35 and a maturity of 1 year. [6]

B6. An environmental technology company is considering an investment in a new patented product that can help industries comply with an environmental regulation that is under consideration by the government. Whereas the regulation is expected to be passed two years from now, many multinational companies based in the country are beginning to comply because of similar regulations already in place in other parts of the world. The company forecasts the present value of the expected future cash flows and the investment (product development and launch) cost to be $400 million each. The annual volatility of the logarithmic returns for the future cash flows is estimated to be 30% for the next two years and is expected to decrease to 20% at the point when the regulations go into effect. The annual continuous risk-free rate for the next four years is 5%. What is the value of this project?

B7. (a) Suppose that you own 5,000 shares worth $25 each. How can put options be used to provide you with insurance against a decline in the value of your holding over the next 4 months? [4]

(b) When first issued, a stock provides funds for a company. Is the same true of a stock option? Discuss. [4]

(c) Explain why a futures contract can be used for either speculation or hedging. [4]

(d) The current price of a stock is $94, and 3-month European call options with a strike price of $95 currently sell for $4.70. An investor who feels that the price of the stock will increase is trying to decide between buying 100 shares and buying 2,000 call options (= 20 contracts). Both strategies involve an investment of $9,400. What advice would you give? How high does the stock price have to rise for the option strategy to be more profitable? [8]

B8. (a) With modern high frequency data, many trades occur during the same second, and hence have the same time stamp. When modelling the durations of time-between-trades, these observations do not add any information. Describe at least four ways to deal with such concurrent observations. [5]

(b) The Weibull density is given as

$$f(x, \alpha, \beta) = \alpha \beta^{-2} x^{\alpha-2} exp\left(-\frac{x^{\alpha}}{\beta}\right), \quad \alpha > 0, \ \beta > 0, \ x \geq 0$$

Derive the hazard function for this density. [5]

(c) Discuss the advantage of the Weibull density over the Exponential density, when modelling durations. What is the main disadvantage of using the Weibull density, leading some researchers to use the Generalized Gamma density instead? [5]

(d) For high frequency duration data, the observed intra-day durations are adapted for intra-day periodicity by dividing through by e.g. the average duration in each 5-minute time interval.

GARCH models on the other hand tend to be estimated on only daily return data. Propose a sensible transformation of intra-day return data, such that these could also be used for estimating a GARCH model. [5]

B9. (a) The price of a stock is $40. The price of a 1-year European put option on the stock with a strike price of $30 is quoted as $7 and the price of a 1-year European call option on the stock with a strike price of $50 is quoted as $5. Suppose that an investor buys 100 shares, shorts 100 call options, and buys 100 put options. Draw a diagram illustrating how the investor's profit or loss varies with the stock price over the next year. How does your answer change if the investor buys 100 shares, shorts 200 call options, and buys 200 put options?. [10]

(b) 'If a company does not do better than its competitors but the stock market goes up, executives do very well from their stock options. This makes no sense." Discuss this viewpoint. Can you think of alternatives to the usual employee stock option plan that take the viewpoint into account. [10]

END OF QUESTION PAPER