NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF ENGINEERING

DEPARTMENT OF ELECTRONIC ENGINEERING

CONTROL ENGINEERING

EEE3241

Examination Paper

March 2025

This examination paper consists of 6 pages

Time Allowed : 3 hours

Special Requirements : Laplace Transform Tables

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External Examiners :

INSTRUCTIONS TO CANDIDATES.

1. In **Section A-** attempt all **four** questions each **fifteen** marks.
2. In **Section-B** attempt any **two** questions each **twenty** marks.
3. Start the answers for each question on a fresh page.
4. Marks will only be awarded for answers that directly relate to the questions asked.

MARK ALLOCATION

	QUESTION	MARKS
SECTION-A	1.	15
	2.	15
	3.	15
	4.	15
SECTION-B	5	20
	6	20
	7	20
TOTAL	6	100

Section A

Question 1

1) What do you mean by a transfer function? Write an example of a typical transfer function.
2) Derive the amplitude ratio and phase angle for a simple first order system.
3) Compare between Feedback control and Feed forward control.
4) Elucidate critically damped and undamped system.
5) Define offset with respect to a controller.
6) Explain adaptive control scheme.
7) Differentiate between discrete and continuous systems.

Two [2] marks each, except for 2) with three [3] marks

Question 2

(a) Apply the Routh-Hurwitz criterion to the characteristic equation

$$s^5 + 2s^4 + 6s^3 + 12s^2 + 8s + 16 = 0$$ [6]

(b) Obtain the value of K for which the system characterized by the following characteristic equation exhibits pureoscillations. What is the frequency of these oscillations?

$$s^3 + 22s^2 + 9s + K = 0$$ [9]

Question 3

Sketch the root loci of the system in Figure 3. The gain K is assumed positive. [15]

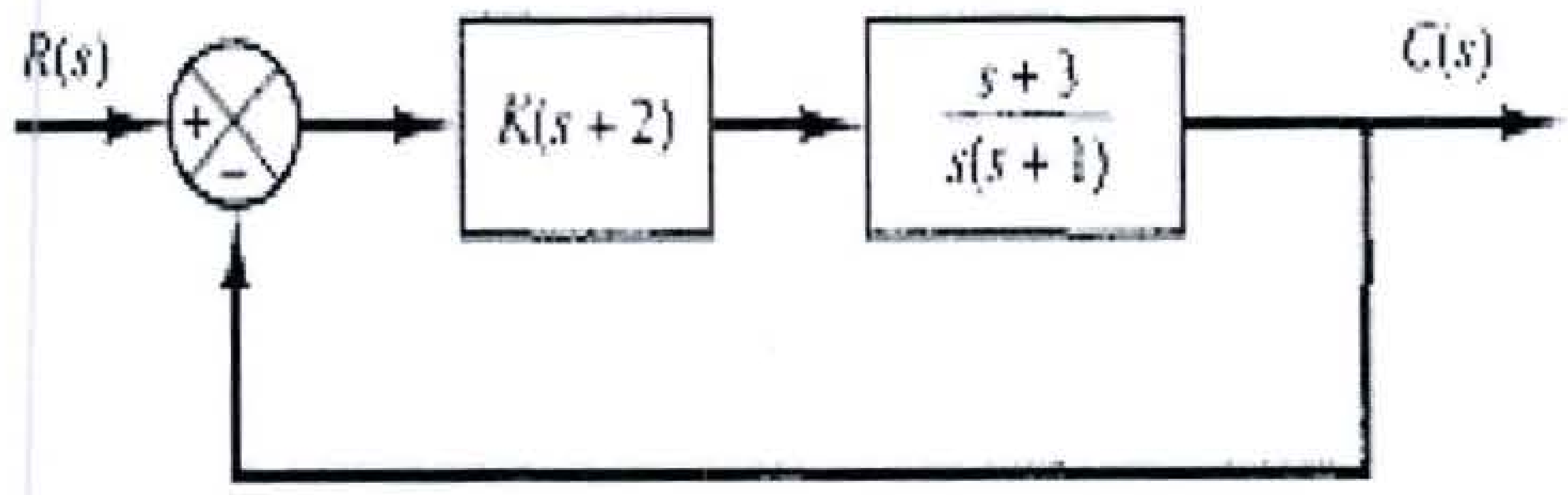

Figure 3

Question 4

From the block diagram shown in Figure 4, determine the overall transfer function. [15]

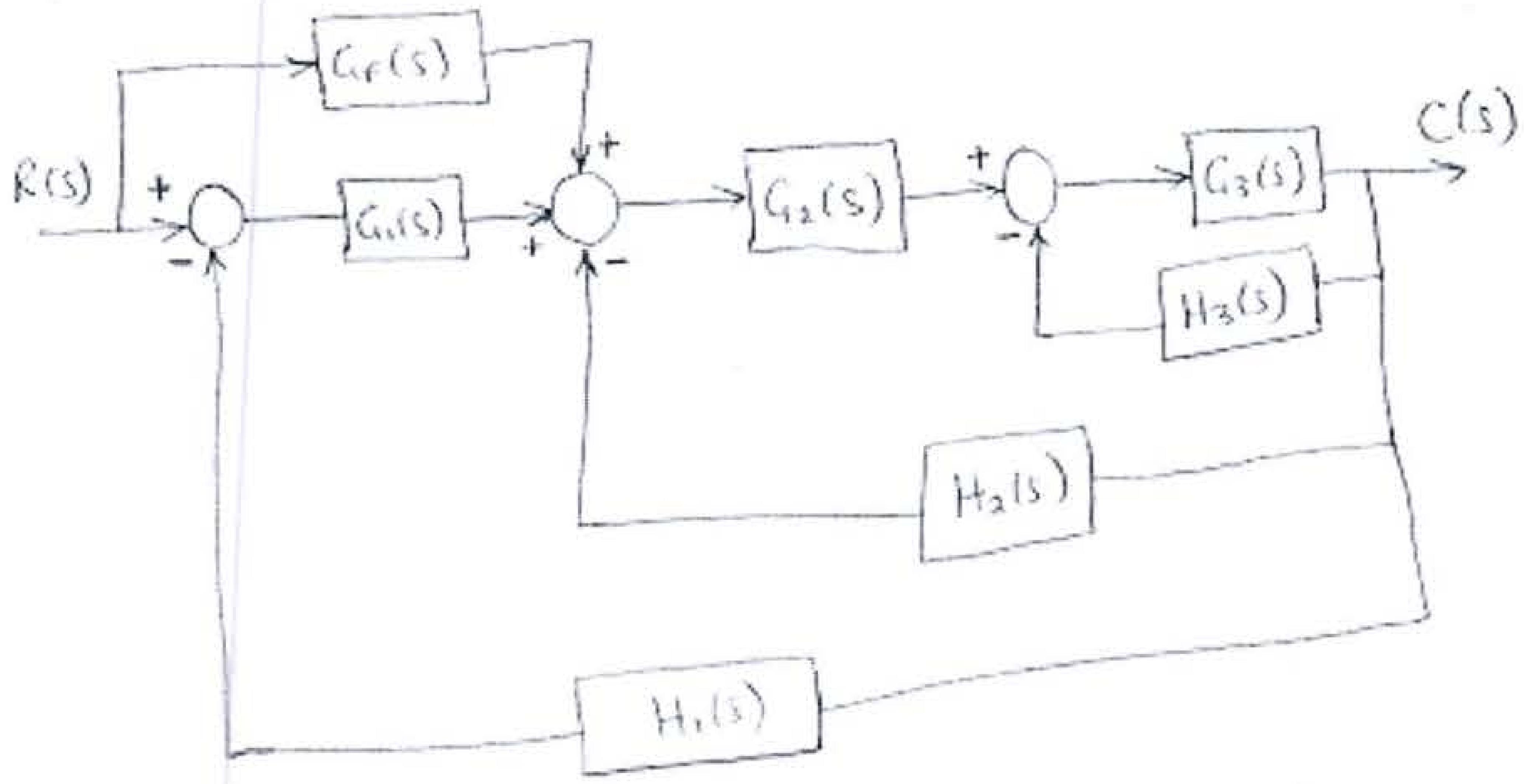

Figure 4

Section B

Question 5

a) Derive the transfer function for a single tank liquid level system [10]

b) Level control systems of the type shown in Figure 5 are commonly used in many processes and chemical industries. In this system the level, h, is measured by the level transducer and its output is used to control a servo motor controlling the valve position which in turn adjusts the inlet flow rate. Determine the equations of motion for the system and determine the dynamic behavior. [10]

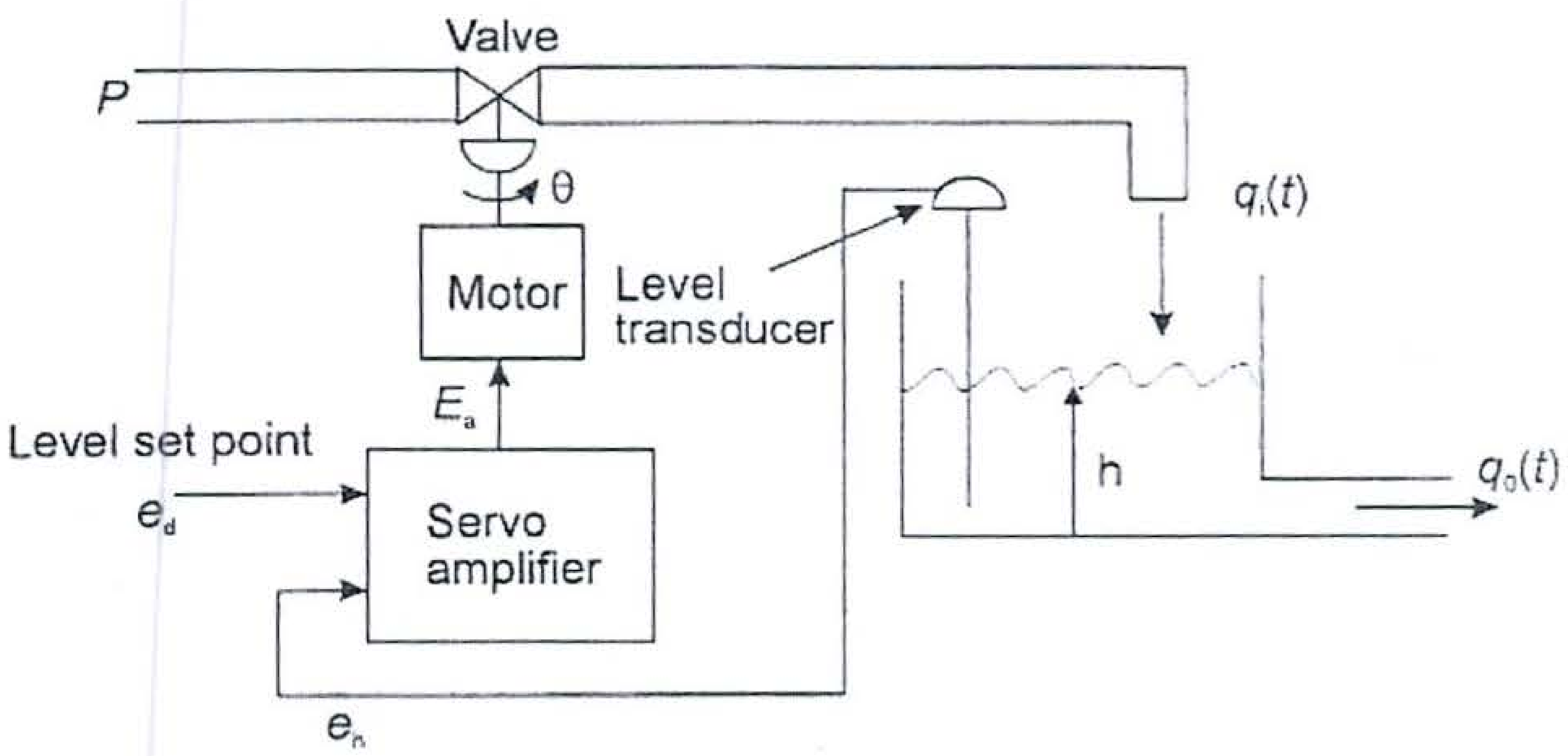

Figure 3.

Question 6

a) Derive the dynamic response of first order system to a sinusoidal input. [8]

b) Differentiate between proportional, proportional integral and proportional integral derivative controllers. Also plot the expense of these controllers to a unit step change in error. [12]

Question 7

A spring-mass-damper system has a mass of 20kg, a spring stiffness 8000N/m and a damper with a damping coefficient of 80Ns/m. The system is excited by a constant amplitude harmonic forcing function of the form $(t) = 160 \sin \omega t$.

a) Determine the system transfer function relating $F(t)$ and $x(t)$ and calculate values of ωn and ζ.

b) What are the amplitudes of the vibration when ω has values of 1.0, 20 and 50rad/s?

c) Find the value of damping coefficient to give critical damping and hence, with this value, determine theamplitudes of vibration for the angular frequencies specified in part (b). [20]

Table of Laplace Transforms

	$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$		$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	$\mathbf{e}^{at}$	$\frac{1}{s-a}$
3.	$t^n,\quad n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	$t^p, p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}},\quad n=1,2,3,\ldots$	$\frac{1\cdot3\cdot5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$\mathbf{e}^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$\mathbf{e}^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$\mathbf{e}^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$\mathbf{e}^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n\mathbf{e}^{at},\quad n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$ Heaviside Function	$\frac{\mathbf{e}^{-cs}}{s}$	26.	$\delta(t-c)$ Dirac Delta Function	$\mathbf{e}^{-cs}$
27.	$u_c(t)f(t-c)$	$\mathbf{e}^{-cs}F(s)$	28.	$u_c(t)g(t)$	$\mathbf{e}^{-cs}\mathfrak{L}\{g(t+c)\}$
29.	$\mathbf{e}^{ct}f(t)$	$F(s-c)$	30.	$t^nf(t),\quad n=1,2,3,\ldots$	$(-1)^nF^{(n)}(s)$
31.	$\frac{1}{t}f(t)$	$\int_s^{\infty}F(u)\,du$	32.	$\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\frac{\int_0^T \mathbf{e}^{-st}f(t)\,dt}{1-\mathbf{e}^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$			

Table Notes

1. This list is not a complete listing of Laplace transforms and only contains some of the more commonly used Laplace transforms and formulas.

2. Recall the definition of hyperbolic functions.

$$\cosh(t) = \frac{\mathbf{e}^{t} + \mathbf{e}^{-t}}{2} \qquad \sinh(t) = \frac{\mathbf{e}^{t} - \mathbf{e}^{-t}}{2}$$

3. Be careful when using "normal" trig function vs. hyperbolic functions. The only difference in the formulas is the "+ a^2" for the "normal" trig functions becomes a "- a^2" for the hyperbolic functions!

4. Formula #4 uses the Gamma function which is defined as

$$\Gamma(t) = \int_0^\infty \mathbf{e}^{-x} x^{t-1}\, dx$$

   If n is a positive integer then,

$$\Gamma(n+1) = n!$$

   The Gamma function is an extension of the normal factorial function. Here are a couple of quick facts for the Gamma function

$$\Gamma(p+1) = p\Gamma(p)$$

$$p(p+1)(p+2)\cdots(p+n-1) = \frac{\Gamma(p+n)}{\Gamma(p)}$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$