NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF ENGINEERING

DEPARTMENT OF ELECTRONIC ENGINEERING

MODERN CONTROL ENGINEERING

EEE 5142

Final Examination Paper

December 2024

This examination paper consists of 6 pages

Time Allowed: 3 hours

Total Marks: 100

Special Requirements: Table of Laplace and Z Transform pairs

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INSTRUCTIONS

1. The question paper comprises of section **A** and section **B**.
2. Section **A** carries a total of 60 marks, and each question from section **B** carries 20 marks.
3. Answer **ALL** questions in section **A** and **any TWO** questions from section **B.**
4. Show your steps clearly in any calculation.
5. Start the answers for each question on a fresh page.

SECTION A (Answer **all** questions).

Question A1

For the following dynamic system:

$$\dot{\bar{x}} = \begin{bmatrix} -3 & 1 & 0 \\ -2 & 0 & \alpha \\ 1 & 0 & 0 \end{bmatrix} \bar{x} + \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} u$$

$$y = [1 \quad 0 \quad 0]\bar{x}$$

where $\alpha = 0$,

a) determine the transfer function $\frac{Y(s)}{U(s)}$ of the system [9 marks]

b) what are the eigenvalues of the system? [3 marks]

Question A2

For the following dynamic system,

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u$$

$$y = \bar{C}\bar{x}$$

where $\bar{A} = \begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix}$, $\bar{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\bar{C} = [1 \quad 2]$

a) Determine the transition matrix $\bar{\Phi}(t)$ for the system. [7 marks]

b) If the states $\overline{x(0)} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, determine the time response of the output $y(t)$ [3 marks]

Question A3

For the following dynamic system determine the matrix $\bar{\mathbf{k}}$ for a state feedback system with poles at $s = -4, -6$

$$\bar{A} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \qquad \bar{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad \bar{C} = [1 \quad 0]$$

[9 marks]

Question A4

In the system shown in figure QA4, $D(z) = \frac{3-3z^{-1}+z^{-2}}{1-z^{-1}}$, $G(s) = \frac{e^{-s}}{s+1}$ and $H(s) = \frac{1}{s}$. Determine the transfer function for the system. [13 marks]

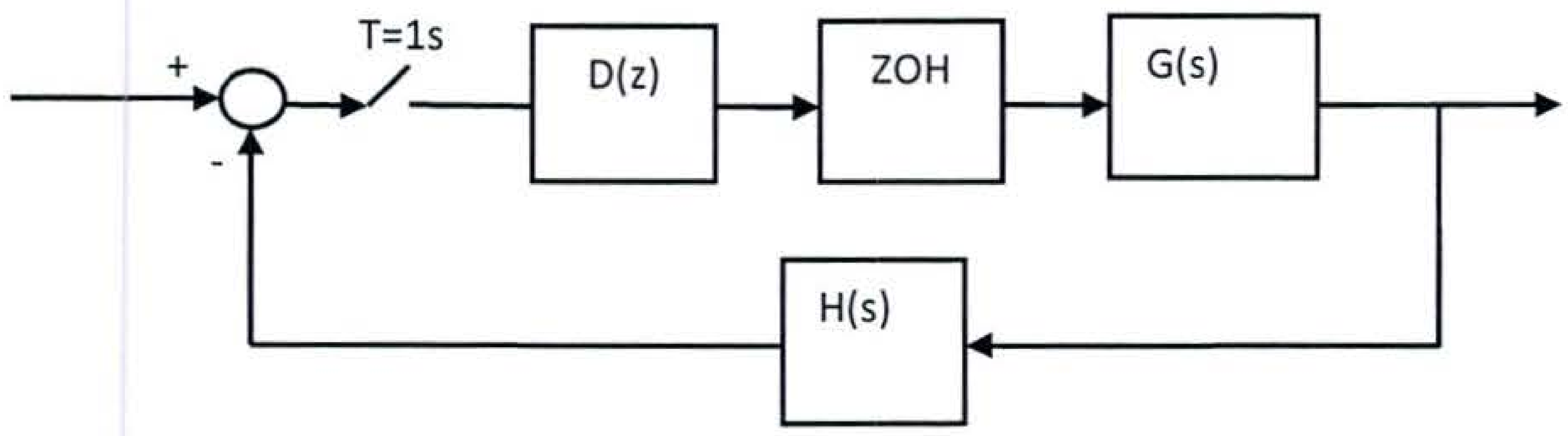

Figure QA4

Question A5

The transfer function of a discrete system is $G(z) = \frac{z^2-2z+2}{z^3+2.1z^2+1.6z-0.4}$. Investigate the stability of the system. [8 marks]

Question A6

In a unity feedback sampled-data system, the Z transfer function of the process preceded by a ZOH is given $G(z) = \frac{1.264}{z^2(z-0.368)}$. The digital control algorithm is $u(k) = 2e(k) - 0.6u(k-1) - 0.4u(k-2)$.

Find

(a) the steady state error for a step input

(b) the steady state error for a ramp input

[8 marks]

SECTION B (Answer **any two** questions only).

Question B1

For the following dynamic system, $\dot{x} = \bar{A}\bar{x} + \bar{B}\bar{u}$ and $y = \bar{C}\bar{x}$, where $\bar{A} = \begin{bmatrix} 0 & 2 \\ 0 & 3 \end{bmatrix}$; $\bar{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $\bar{C} = [1 \quad 0]$,

Determine a suitable state-feedback vector $\bar{\boldsymbol{k}}$ and forward path gain $\boldsymbol{K}$ to ensure closed-loop poles at $s = -1 \pm j$ and steady-state gain equal to 5. [20 marks]

Question B2

For the sampled data system in figure QB2, $T = 0.5$ seconds and $D(z)$ is a ***PI*** controller, i.e. $D(z) = k\frac{z-\lambda}{(z-1)}$ with k and λ are to be determined to give 8 samples per damped oscillation and dominant poles with a damping ratio $\zeta = 0.5$.

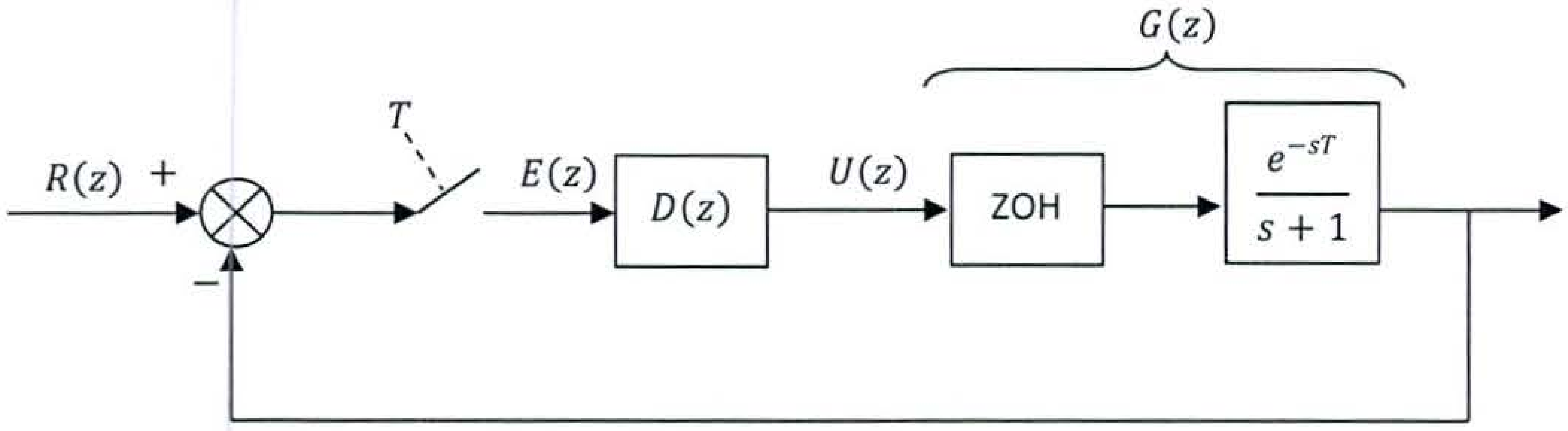

Figure QB2

Design the controller D(z) [20 marks]

Question B3

The system shown in Figure QB3 is to have dominant poles with a damping ratio of 0.5 and a settling time of 2 seconds.

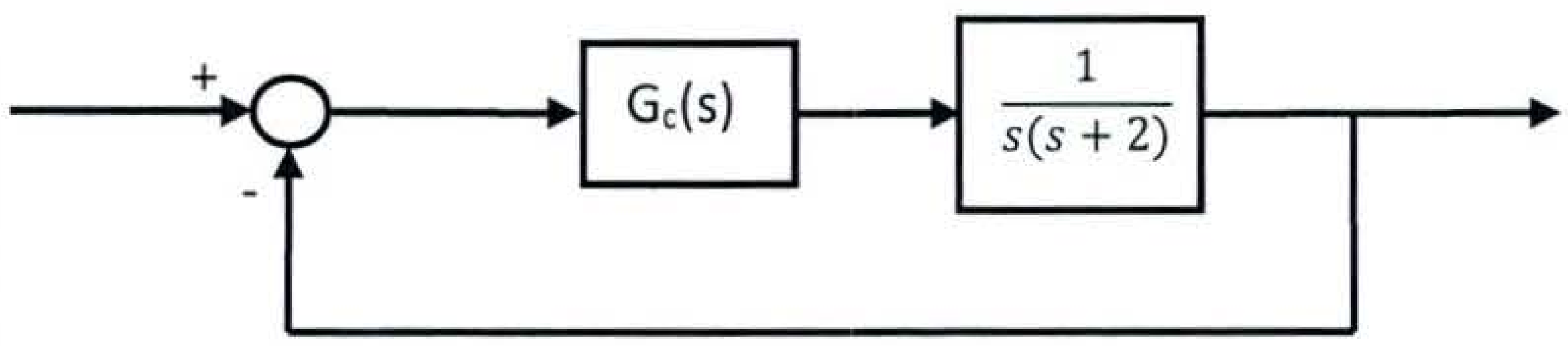

Figure QB3

a) What are the values of ω_n and ω_d? [6 marks]
b) To turn this system into a discrete one, choose a sampling period which will sample 10 times each damped period and give the corresponding ZOH approximation. [5 marks]
c) A suitable controller is then given by $G_c(s) = \frac{20(s+2)}{s+7}$. Turn this into an equivalent digital algorithm using a suitable transform method. [7 marks]
d) What are the advantages of Transform methods in designing discrete controllers? [2 marks]

Appendix A: List of Formulae

State space TF: $G(s) = \frac{\bar{C}adj[s\bar{I} - \bar{A}]\bar{B}}{det[s\bar{I} - \bar{A}]}$

Ackerman's Formulae: $\bar{k} = [0 \quad \ldots \quad 1]\bar{Q}^{-1}\Delta_d(\bar{A})$

Settling time: $T_s = \frac{4}{\zeta\omega_n}$

Damped frequency: $\omega_d = \omega_n\sqrt{1 - \zeta^2}$

Position of dominant poles: $|z| = e^{-\left(\frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \times \frac{\omega_d}{\omega_s}\right)}$ and $\angle z = \pm 2\pi\frac{\omega_d}{\omega_s}$

ZOH approximation: $= \frac{1+Ts}{1-Ts}$

Bilinear Method: $D(z) = G_c(s)_{|s=\frac{2(1-z^{-1})}{T(1+z^{-1})}}$

Backward Difference Method: $D(z) = G_c(s)_{|s=\frac{1-z^{-1}}{T}}$

Appendix B: Table of Laplace and z-Transform Pairs

f(t)	F(s)	f(k) = f(kT)	F(z)
1. impulse, $\delta(t)$	1	$\delta(k)$	1
2. unit step, $u(t)$	$\frac{1}{s}$	$u(k) = 1$	$\frac{z}{z-1} = \frac{1}{1-z^{-1}}$
3. shifted impulse, $\delta(t-nT)$	e^{-nsT}	$\delta(kT-nT)$	z^{-n}
4. unit ramp, t	$\frac{1}{s^2}$	kT	$\frac{Tz}{(z-1)^2}$
5. parabola, t^2	$\frac{2}{s^3}$	$(kT)^2$	$\frac{T^2 z(z+1)}{(z-1)^3}$
6. exponential, $e^{\pm at}$	$\frac{1}{s \mp a}$	$e^{\pm akT}$	$\frac{z}{z-e^{\pm aT}} = \frac{1}{1-e^{\pm aT}z^{-1}}$
7. $e^{-at} f(t)$	$F(s+a)$	$e^{-akT} f(kT)$	$F(z\, e^{aT})$
8.		c^k	$\frac{z}{z-c} = \frac{1}{1-cz^{-1}}$
9. $t\, f(t)$	$\frac{-dF(s)}{ds}$	$kT\, f(kT)$	$-Tz\frac{dF(z)}{dz}$
10. $\frac{1}{t} f(t)$	$\int_s^{\infty} F(s')\, ds'$	$\frac{1}{kT} f(kT)$	$-\frac{1}{T}\int \frac{F(z)}{z} dz$
11. $\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$	$\sin \omega kT$	$\frac{z \sin \omega T}{z^2 - 2z\cos \omega T + 1}$
12. $\cos \omega t$	$\frac{s}{s^2+\omega^2}$	$\cos \omega kT$	$\frac{z(z-\cos \omega T)}{z^2 - 2z\cos \omega T + 1}$
13. $e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-akT} \sin \omega kT$	$\frac{z e^{-akT} \sin \omega T}{[z-e^{(-a+j\omega\omega)}][z-e^{(-a-j\omega\omega)}]}$
14. $e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-akT} \cos \omega kT$	$\frac{z(z-e^{-akT}\cos \omega T)}{[z-e^{(-a+j)T}][z-e^{(-a-j\omega\omega)}]}$
15. $\frac{e^{-at}-e^{-bt}}{b-a}$	$\frac{1}{(s+a)(s+b)}$		
16. $\frac{\omega}{\sqrt{(1-\zeta^2)}} e^{-\zeta\omega t} \sin\left[\omega\sqrt{(1-\zeta^2)}\, t\right]$	$\frac{\omega^2}{s^2+2\zeta\omega s+\omega^2}$		
17. time advance, $f(t+T)$	$F(s)\, e^{+sT}$	$f(kT+T)$	$z[F(z) - f(0)]$
18. double time advance, $f(t+2T)$	$F(s)\, e^{+2sT}$	$f(kT+2T)$	$z^2[F(z)-f(0)] - zF(T)$
19. general time advance, $f(t+nT)$	$F(s)\, e^{+nsT}$	$f(kT+nT)$	$z^n F(z) - z^n f(0) - z^{-1} f(T) \cdots - zf(nT-T)$
20. general time retard, $f(t-nT)$	$F(s)\, e^{-nsT}$	$f(kT-nT)$	$z^{-n}F(z)$, one sided

END OF PAPER