NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION

DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION

BACHELOR OF EDUCATION

LINEAR MATHEMATICS

PST1130

Main Examination Paper

NOVEMBER 2024

This examination paper consists of 4 pages

Time Allowed: 3 hours

Total Marks: 100

Special Requirements: None

Examiner's Name: Sesilani Nkomo

External Examiner: Dr Sunzuma G

INSTRUCTIONS

1. This paper consists of **NINE** questions. Answer **ALL** questions in section A and at **most three** questions in section B.
2. All answers should be presented in good style.
3. Begin each full question on a new page.

MARK ALLOCATION

QUESTION	MARKS
1-4	40
5.	20
6.	20
7.	20
8.	20
9.	20

Section A [40marks]

Answer **all** questions in this section.

1. If $p = -4 + 3i$ and $q = -1 + \sqrt{3}i$
   a) Calculate the modulus of
      i) p [2]
      ii) q [2]
   b) Find
      i) The argument of q [2]
      ii) pq^2 in the form $a + bi$ [3]
      iii) $\frac{p}{q}$ in the form $a + bi$. [3]

2. Matrix $A = \begin{pmatrix} -1 & -1 & 1 \\ 0 & -1 & 3 \\ 2 & 1 & 2 \end{pmatrix}$, Matrix $B = \begin{pmatrix} -3 & 2 & -1 \\ 2 & -1 & 3 \\ -1 & 1 & 1 \end{pmatrix}$ and

   Matrix $C = \begin{pmatrix} 4 & 3 & 1 \\ 2 & -1 & 3 \\ 1 & 0 & 1 \end{pmatrix}$
   a) Find $\boldsymbol{AB}$. [3]
   b) Show that matrix $\boldsymbol{C}$ is singular. [2]
   c) Find the
      i) determinant of $\boldsymbol{A}$. [2]
      ii) inverse of $\boldsymbol{A}$. [5]
   d) Hence or otherwise, solve the simultaneous equations

      $-x - y + z = 1$
      $-y + 3z = 8$ [4]
      $2x + y + 2z = 8$

3. The points A, B and C have position vectors$\boldsymbol{a} = \boldsymbol{i} - 2\boldsymbol{j} + p\boldsymbol{k}$, $\boldsymbol{b} = q\boldsymbol{i} + 5\boldsymbol{j} + 6\boldsymbol{k}$ and $\boldsymbol{c} = 5\boldsymbol{i} + 7\boldsymbol{j}$ respectively relative to the origin. If $\overrightarrow{\boldsymbol{AB}} = -5\boldsymbol{i} + 7\boldsymbol{j} + 3\boldsymbol{k}$ find the
   a) values of p and q. [2]
   b) exact length of AC. [2]
   c) acute angle BAC. [3]

4. Solve the equation $z^2 + 3z + 5 = 0$, giving the answers in the form $x + iy$. [5]

Section B [60marks]

Answer **at least 3** questions in this section.

5. a) On a single diagram shade the region defined by the inequalities

$\frac{\pi}{6} \leq \arg(z-4) \leq \frac{\pi}{4}$ and $|z-4| \leq 4$. [3]

b) Given the polynomial $f(x) = x^3 - 5x^2 + 16x - 30$,

i) Show that $z = 1 + 3i$ is a root of the equation $f(x) = 0$. [2]

ii) Find the other two roots of $f(x) = 0$. [3]

c) Solve the equation

$z^3 = -5 + 12i$. [6]

d) Use de Moivre's theorem to show that

$Cos\ 6\theta \equiv 32Cos^6\theta - 48Cos^4\theta + 18Cos^2\theta - 1$. [6]

6. a) Solve the system

$$\begin{aligned} 3y + 2x &= z + 1 \\ 3x + 2z &= 8 - 5y \\ 3z - 1 &= x - 2y \end{aligned}$$

using Cramer's rule. [7]

b) For the system

$$\begin{aligned} x + 5y + \alpha z &= 0 \\ -2x + 6y + 2z &= 0 \\ \alpha x - y + z &= 0 \end{aligned}$$

i) find the values of α for which the system has non-trivial solutions. [4]

ii) find the non-trivial solutions, if $\alpha = 3$. [4]

c) Use the Gaussian elimination to solve the following system of equations

$$\begin{aligned} 2x + 5y - 8z &= 4 \\ x + 2y - 3z &= 1 \\ 3x + 8y - 13z &= 7 \end{aligned}$$

. [5]

7. a) Find the value of α so that vectors **u**=$(2, 3\alpha, -4,\ 1, 5)$ and **v**=$(6, -1, 3,\ 7, 2\alpha)$ are orthogonal. [3]

b) The line L_1 has equation $r = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ and the plane π_1 passes through the points $A,\ B$ and C with coordinates $(2;\ -1;\ 3)$, $(4;\ 2;\ -5)$ and $(-1;\ 3;\ -2)$ respectively.

Find the

i) cartesian equation of π_1. [5]

ii) acute angle between the plane π_1 and the line L_1. [3]

c) The plane π_2 has equation $\boldsymbol{r} \cdot \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} = 6.$

The line L_2 lies in the plane π_2 and is perpendicular to L_1. The line L_2 passes through the point $(4;\ 2;\ 1)$.

Find the

i) vector equation of L_2. [4]

ii) vector equation of the line of intersection of the planes π_1 and π_2. [5]

8. a) Find the eigenvalues and corresponding eigenvectors of the following 3×3 matrix $A = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}$. [8]

b) Find the solution of $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ of the system

$\begin{pmatrix} 3 & 1 & 6 \\ -6 & 0 & -16 \\ 0 & 8 & -17 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 17 \end{pmatrix}$ using LU decomposition. [6]

c)If A=$\begin{pmatrix} 2 & -1 & 1 \\ 4 & -3 & 0 \\ -3 & 3 & 1 \end{pmatrix}$, use Cayley-Hamilton theorem to find A^{-1}. [6]

END OF QUESTION PAPER