NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION

DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION

LINEAR MATHEMATICS PST1130

Examination Paper

DECEMBER 2024

This examination paper consists of 3 pages

Time Allowed: 3 hours

Total Marks: 100

Special Requirements: None

Examiner's Name: Sesilani Nkomo

INSTRUCTIONS

1. This paper consists of **six** questions. Answer **at most five** questions.
2. All answers should be presented in good style.
3. Begin each full question on a new page.

MARK ALLOCATION

QUESTION	MARKS
1.	20
2.	20
3.	20
4.	20
5	20
6	20

1. a) Simplify $\begin{pmatrix} 3 & 7 & 5 \\ 2 & -1 & 4 \end{pmatrix}\begin{pmatrix} 3 & -1 & 1 \\ 2 & 4 & -6 \\ 7 & 5 & 6 \end{pmatrix}$ [7]

b) Find x such that $\begin{vmatrix} 2-x & 1-x & 3 \\ 2-x & 3 & 1-x \\ 2 & 1-x & 3-x \end{vmatrix} = 0$ [13]

2. Simplify

a) $(-3+2i)(8-7i)$ [5]

b) $\frac{6+5i}{-4-3i}$ [5]

c) $(Cos\frac{\pi}{6} + iSin\frac{\pi}{6})^3$ [5]

d) $\frac{(1+i)^4}{(2-2i)^3}$, giving your answer in the form $a + bi$ [5]

3. a) If $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$, show that $A^2 - 4A - 5I = 0$ [6]

b) Given that $M = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix}$

find i) Adj (M) [10]

ii) M^{-1} [4]

4. a) Solve the following system of equations

$$2x + 6y + 2z = 2$$
$$-3x - 8y = 2$$
$$4x + 9y + 2z = 3$$
[10]

b) Solve the following system of equations using Cramer's rule

$$x_1 + 2x_2 + 3x_3 = 5$$
$$2x_1 + 5x_2 + 3x_3 = 3$$
$$x_1 + 8x_3 = 17$$
[10]

5. a) Find the magnitude of $\frac{1}{2}\boldsymbol{i} - \frac{15}{4}\boldsymbol{j} + \frac{11}{5}\boldsymbol{k}$ [5]

b) Given $\boldsymbol{a} = 2\boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k}$ and $\boldsymbol{b} = 3i - \boldsymbol{2}j + 5k$. Find

i) $\boldsymbol{a}.\boldsymbol{b}$ [5]

ii) the angle between direction vectors [5]

iii) $6(5\boldsymbol{a} - 2\boldsymbol{b})$ [5]

6. a) Write the vector $\boldsymbol{V} = (1, -2, 5)$ as a linear combination of the vectors $(1, 1, 1)$, $(1, 2, 3)$, $(2, -1, 1)$ [10]

b) Find the conditions on a, b and c so that $(a, b, c) \in R^3$ belongs to the space generated by

$u_1 = (2, 1, 0)$, $u_2 = (1, -1, 2)$ and $u_3 = (0, 3, -4)$ [10]