



NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY  
FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION  
DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION  
BACHELOR OF EDUCATION-PHYSICS

CALCULUS 1 PST 1131

Main Examination Paper

December, 2024

This examination paper consists of 4 pages

Time Allowed: 3 hours

Total Marks: 100

Special Requirements: None

Examiner's Name: Mrs K Moyo

External Examiner: Dr Sunzuma G

**INSTRUCTIONS**

Answer **ALL** questions in section A and **THREE** questions in section B

**MARK ALLOCATION**

QUESTION	MARKS
A1	9
A2	8
A3	11
A4	12
B5	20
B6	20
B7	20
B8	20
TOTAL	100

**SECTION A (40 marks)**

Candidates may attempt ALL questions being careful to number them A1 to A4.

**A1.** Let  $f(x) = x^2 + 2x - 1$  for all  $x$ . Evaluate:

(a)  $\frac{f(x+h)-f(x)}{h}$  [5]

(b)  $f(x + 1)$  [4]

**A2.** Specify the domain and the range of the following functions using interval notation

(a)  $f(x) = \frac{1}{(x-2)(x-3)}$  [4]

(b)  $h(x) = \begin{cases} x + 1 & -1 < x < 1 \\ 2 & 1 \leq x \end{cases}$  [4]

**A3.** Solve the following inequalities

(a)  $(2x - 1)(x + 1) \geq 0$  [3]

(b)  $\frac{(x-2)^2-8}{5-4x} > 1$  [4]

(c)  $|3x - 2| \geq 1$  [4]

**A4.** (a) Find  $\lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$  for the limits of the following functions

(i)  $f(x) = 3x - 1$  [4]

(ii)  $f(x) = 4x^2 - x$  [4]

(b) Find the derivative function  $f$  such that  $f(x) = \frac{x^2+x-2}{x^3+4}$  [4]

**SECTION B (60 marks)**

Candidates may attempt THREE questions being careful to number them B5 to B8.

**B5.** (a) Use the basic definition of a derivative to find the limit of the following functions:

(i)  $f(x) = 2x^3 + 3x - 1$  [4]

(ii)  $f(x) = 2x - 5$  [3]

(iii)  $f(x) = \cos x$  [5]

(b) Evaluate  $\int_0^1 \sqrt{5x + 4} dx$  [8]

**B6.**(a) Find the absolute maximum and minimum values of the following functions at the given intervals

(i)  $f(x) = x^3 - 5x^2 + 3x + 1$  on  $[0,1]$  [6]

(ii)  $\frac{x^2+3}{x+1}$  on  $[0,3]$  [8]

(b) If  $f(x) = 2x$  and  $g(x) = \frac{1}{x-1}$  find all the solutions of the equation

$f \circ g(x) = g \circ f(x)$  [6]

**B7.** (a) Show that  $\lim_{x \rightarrow \infty} \left(1 + \frac{3}{x}\right)^x = e^3$ . [8]

(b) The function  $f$  is given by  $f(x) = 1 - \frac{1}{x}$ . Find:

i)  $f^{-1}(4)$  [6]

ii) the value of  $x$  for which  $f^{-1}(x) = 2$ . [6]

**B8.** (a) Prove by mathematical induction that

$1.2 + 2.3 + 3.4 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$  for all integers  $n \geq 1$ . [8]

(b) Find the following anti-derivatives

(i)  $\int (x^2 + 3x - 5)^3 (2x + 3) dx$  [6]

(ii)  $\int x^2 \sqrt{x + 2} dx$  [6]