NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION

DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION

MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

PST 1133

MAIN EXAMINATION PAPER

DECEMBER 2024

This Examination Paper consists of 4 pages

Time Allowed: **3 hours**

Total Marks: **100**

Special Requirements: **NONE**

Internal Examiner: **Ms D Moyo**

External Examiner: **Dr B Moyo**

INSTRUCTIONS

1. Answer ALL questions in **Section A** and any **THREE (3)** questions in **SECTION B.**

2. Each question carries 25 marks

MARK ALLOCATION

QUESTION	MARKS
1.	25
2.	25
3.	25
4.	25
5.	25
TOTAL	100

SECTION A (40marks)

1. Given that X = {a, b, c} and Y = {c, d, e}, find
   a) $Y + X$ [3marks]

   b) $X - Y$ [2marks]

   c) $P(X)$ [2marks]

2. Let A ={2,4,6,8}.Define the relation R on A by $\{x, y\} \in R$ iff x divides y. [4marks]

3. Given $P(x): x > 3$
   What are the truth values of
   a) $P(4)$ [2 marks]

   b) $P(2)$ [2 marks]

4. Construct a truth table to show that;
   $(p \rightarrow q) \leftrightarrow (\neg p \vee q)$ is a tautology. [6 marks]

4. Given a function such that $f(x) = 4x - 7$;

   a) Solve $f(x) = 17$ [3marks]

   b) $f^{-1}(x)$ [3marks]

5. In how many ways can 5 boys and 4 girls be arranged on a bench if ;
   a) There are no restrictions. [2marks]

   b) Boys and girls alternate. [3marks]

6. A committee of 5 people is to be chosen from a group of 6 men and 4 women.
   How many committees are possible if;
   a) There must be 3 men and 2 women. [3marks]

   b) The committee is composed of men only. [2marks]

   c) The committee must have a majority of women. [3marks]

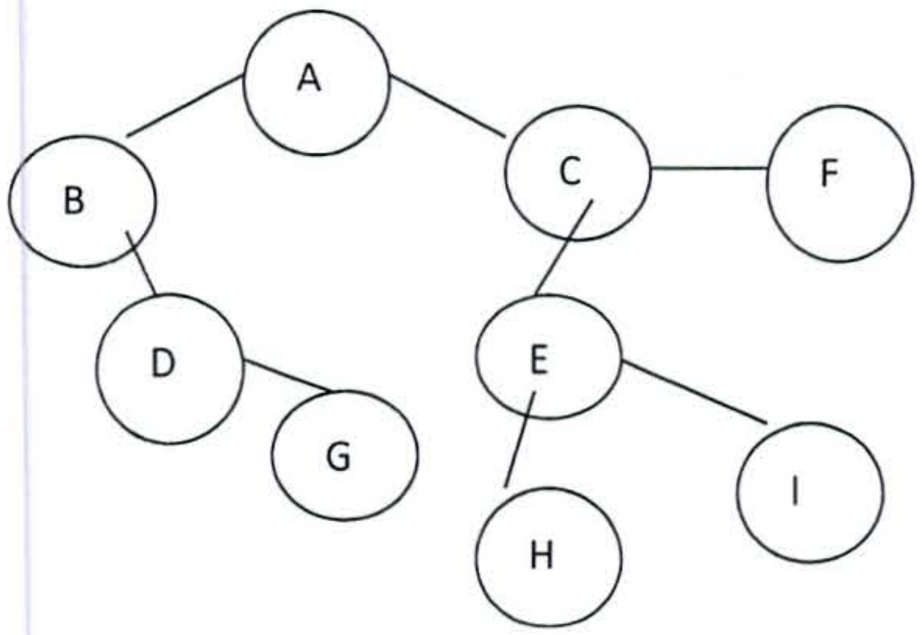
SECTION B(60marks)

7. Given that A, B, C, D, E, F, G, H and I are nodes, where A is the root node as shown in the binary tree below. What is the output when the tree is traversed in:

a) Pre order [3marks]

b) In order [3marks]

c) Post order [3marks]



d) Draw a binary tree where the nodes would be listed in pre order traversal as A B D C E F. [3 marks]

e) Given that $g(x) = x^3 + 7x - x + 1$ and $f(x) = 4x^2$.

i. Find $g(f(x))$. [3marks]

ii. Find the inverse function of $g(x) = 2x - 7$. [4marks]

8. a) Given that A and B are sets. Prove that:

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ [5marks]

b) Consider the set P = {1, 2, 3, 4, 6, 9}. Define a relation R by writing (x, y) $\in$R if and only if x-y is a multiple of 3.

i) Describe R as a subset of P x P. [3marks]
ii) Show that R is an equivalence relation on P. [6marks]

b) Describe briefly the following properties of events;

i. Mutually exclusive events. [3marks]

ii. Independent events. [3marks]

9. a) Let R be the set of real numbers and if $f; R \to R$ be the function defined by $f(x) = 4x + 5$. Show that f is invertible and find f^{-1}. [6marks]

b) A box contains 9 black and 5 red balls. 2 balls are randomly drawn from the box, find,

i. the probability of getting both balls black, when the first ball drawn is replaced. [2marks]

ii. the probability of getting both balls red when the first ball drawn is not replaced. [2marks]

c)Given that $f(x)=3x^2-x-1$ and $g(x)=x-1\frac{1}{2}$. Find

i. $f(x)+\frac{1}{2}g(x)$ [2marks]
ii. $g(x) - f(x)$ [2marks]
iii. $(fOg)(x)$ [2marks]
iv. $(gOf)(x)$ [2marks]
v. $g^{-1}(x)$ [2marks]

10. a) With the aid of examples explain the following terms:
i. Conjunction [2marks]
ii. Disjunction [2marks]
iii. Negation [2marks]
iv. Implication [2marks]

v. Biconditional [2marks]

b) Investigate whether the following is a tautology or not
$p \wedge (q \wedge r) \leftrightarrow ((p \wedge q) \wedge r)$ [4marks]

a) Given that p, q and r are atomic sentences, investigate whether the following compound sentence is a Tautology or not.
$(p \wedge (q \vee r)) \leftrightarrow ((p \wedge q) \vee (p \wedge r))$ [6marks]

END OF PAPER