NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION

DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION

BACHELOR OF SCIENCE EDUCATION (HONOURS) DEGREE IN MATHEMATICS

PROBABILITY THEORY AND STATISTICS   (PST 1233)

Special Examination Paper   **August 2024**

This Examination Paper consists of 4 printed pages

Time Allowed: 3 hours

Total Marks: 100

Special Requirements: Statistical Tables

Internal Examiner: Mr Mkwelie   N

INSTRUCTIONS

1. Candidates should attempt **All** questions in Section A and any other **THREE** questions in section B.
2. Each question should start on a fresh page.
3. Section A carries 40 marks.
4. Each question in Section B carries 20 marks.
5. Omission of essential working will result in loss of marks.
6. Candidates will be penalised for use of wrong mathematical notation.

MARK ALLOCATION

QUESTION	MARKS
A1-6	40
B7	20
B8	20
B9	20
TOTAL	100

Section A [40 Marks]

Attempt **ALL** questions in this section being careful to number them **A1** to **A4**.

A1.(a) (i) Define a mutually exclusive event.　　　　　　　　　　　　　　　[2]

　　　(ii) Construct a probability distribution function of tossing an ubiased die once.　[3]

　(b)　A discrete random variable has a probability density function on the table below.

x	1	2	3	4	5
$P(X = x)$	0.1	0.2	0.3	c	0.1

Find

(i)　　The value of c.　　　　　　　　　　　　　　　　　　　　　　　　　[2]

(ii)　　$P(X \leq 3)$　　　　　　　　　　　　　　　　　　　　　　　　　　[2]

(iii)　　E(X)　　　　　　　　　　　　　　　　　　　　　　　　　　　　　[3]

(iv)　　Var(X)　　　　　　　　　　　　　　　　　　　　　　　　　　　　[3]

A2. A certain Bank offers three possible ways of investing money, A, B and C. The probability that one invests in A is 0.25, in B is 0.65 while in C it is 0.10. The probaility that investment A will lead to profit is 0.5, for B it is 0.3 while for C it is 0.7.

(a) Construct a tree diagram for the possible investments.　　　　　　　　　[3]

(b) Find the probability that investment A will lead to a loss.　　　　　　　[3]

(c) What is the probability of making a loss?　　　　　　　　　　　　　　[3]

A3. The probability that a BSc Mathematics student will pass the Probability Theory examination is 0.6. In a class of 20 students, use the Binomial distribution to find the:

(a) probability only 5 students will pass the exam.　　　　　　　　　　　　[3]

(b) probability that at least 5 students will pass the exam.　　　　　　　　　[3]

(c) number of students expected to pass the exam.　　　　　　　　　　　　[3]

A4. The number of accidents in a week on a stretch of road is known to be follow a Poison distribution with mean 2.1. Find the probability that,

(a) in a given week there is only one accident.　　　　　　　　　　　　　　[3]

(b) In a two week preiod there are 2 accidents.　　　　　　　　　　　　　[3]

(c) When do we use Poison approximation to Binomial distribution?　　　　[2]

SECTION B (60 Marks)

Answer any **THREE** questions in this section being careful to number them B5 to B8.

B5(a) A random variable X has probability density function $f(x)$ where

$$f(x) = \frac{1}{8}x, 0 \leq x \leq 4$$

    (i) Sketch $f(x)$ [2]

    (ii) Find $P(0.3 \leq x \leq 1.8)$ [3]

    (iii) Find the cumulative distributive function X.   [3]

  (b) A computer company has three machine that produce a certain type of computer chip. Machine A produces 1000 chips, machine B 2000 chips and machine C 5000 chips. Of the chips produced by machine A, 20% are found to be defective, by machine B, 5% are found to be defective and by machine C, 10% are found to be defective.

    (i) Construct a probability tree diagram.   [5]

    (ii) Find the probability of a chip being defective.   [3]

    (iii) Find the probability that a chip was produced by machine A given that it is defective.   [3]

B6. (a) The probability that a patient recovers from Covid-19 disease is 0.8. If 15 people are known to have contracted this disease.

    (i)   What is the probability at least 10 survive ?   [6]

    (ii)   State the properties of a Binomial distribution.   [5]

  (b) The continuous random variable X has probability density function

$$f(x) = \frac{1}{36}(9 - x^2) \text{ for } -3 \leq x \leq 3.$$

    (i)   Find the mean and variance of X.   [5]

    (ii)   Calculate $P(X > 2)$.   [4]

B7. .Let X have the Binomial distribution with parameters n and p.

    Find,

        a) the moment generating function(mgf) of X,   [7]

        b) the mean and variance of X using the moment generating function.   [8]

        c) the probability generating function of X.   [5]

  B8. (a) The random variable X has probability function given by

$$P(x = x) = \frac{11-2x}{25}, x = 1,2,3,4,5.$$

    (i)   Construct a table giving the probability distribution of X.   [4]

(ii)   Find $E(X)$ and $\text{Var}(X)$.   [5]

(iii) $\text{Var}(2X - 3)$ [3]

(b) Suppose the price of 2kg sugar is normally distributed with mean \$250.00 and standard deviation of \$150.000. Find the probability that:

(i) the price of sugar is less than \$400.00   [4]

(ii) the price of sugar is between \$150.00 and \$400.00.   [4]

END OF EXAMINATION PAPER