NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION

DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION

PROBABILITY THEORY AND STATISTICS (PST1233)

Examination Paper **December 2024**

This Examination Paper consists of 4 printed pages

Time Allowed: 3 hours

Total Marks: 100

Special Requirements: Statistical tables

Internal Examiner: Mr T Moyo

External Examiner: Dr G Sunzuma

INSTRUCTIONS

1. Answer all questions in section A and any three questions in section B.
2. Each question should start on a fresh page
3. Marks will be allocated as indicated below

MARK ALLOCATION

QUESTION	MARKS
A1	5
A2	7
A3	8
A4	8
A5	12
B6	20
B7	20
B8	20
B9	20
TOTAL	100

Section A (40 marks)

Candidates should attempt ALL questions, being careful to number them A1 to A5

A1. a) Define the term possibility space. (1)

b) Describe any 2 properties of a standard normal probability distribution. (2)

c) Distinguish between Independent and mutually exclusive events. (2)

A2. a) Let X be a random variable with mean 5 and variance 1. Find the lower bound to the probability $P(1 < X < 2)$. (3)

b) When watching a game of Zimbabwe Football League in a bar, you observe someone who is clearly supporting Highlanders Football Club during the game. What is the probability that they were actually born in Bulawayo? We are given that the probability that a randomly selected person in a typical local bar environment is born in Bulawayo is $\frac{3}{10}$ and the chance that a person born in Bulawayo actually supports Highlanders is $\frac{3}{5}$. The probability that a person not born in Bulawayo supports Highlanders is $\frac{1}{10}$. (4)

A3. The distribution for a random variable X is given by $f(x) = \begin{cases} 4e^{-4x} & x \geq 0 \\ 0 & x < 0 \end{cases}$

a) Show that f(x) is a probability density function, (2)

b) Find the mean of the distribution (2)

c) Calculate $P(|X| < 1)$ (3)

A4. A bag contains 4 white and 6 blue balls. Two balls are chosen at random from the bag, without replacement.

a) Draw a tree diagram to illustrate the scenario (2)

b) What is the probability that,

(i) the second ball is red, (2)

(ii) at least one ball is blue. (2)

(iii) first ball is blue given that the third ball is white. (2)

A5. The joint probability function of two random variables X and Y is given in table below

Y \ X	1	2	3
1	1/6	1/12	1/8
2	1/8	1/16	3/16
3	1/24	k	3/16

a) Find the value of k. (1)

b) Find marginal distribution of X and Y. (4)

c) Find Cov(X,Y). (6)

d) Are X and Y independent? (1)

Section B (60 Marks)

Answer any **THREE** questions being careful to number them B6 to B9.

B6. A continuous random variable X has the probability density given by

$$f(x) = \begin{Bmatrix} 2e^{-2x} & x > 0 \\ 0 & x \leq 0 \end{Bmatrix}$$

a) Show that f(x) is a legitimate pdf (3)

b) Find (i) $E(X)$ (4)

(ii) $E(X-1)$ (4)

(iii) $Var(X)$ (4)

c) Find the moment generating function of X (5)

B8. a) Students A, B and C in that order throw a tetrahedral die. The first one to throw a '4' wins. The game continues indefinitely until someone wins. Find the probability that

(i) A wins? (3)

(ii) B wins? (3)

(iii) C wins? (3)

b) Let X be a random variable with probability density function

$$f(x) = \begin{cases} \frac{1}{c\pi} & -\pi < x < \pi \\ 0 & elsewhere \end{cases}$$

a) Find c for which $f(x)$is a legitimate pdf. (3)

b) Find Cumulative Distribution Function(CDF) of X. (5)

c) Calculate the median of X. (3)

B9. The joint probability function of two discrete random variables X and Y is given by $f(x,y) = c(2x + y)$ where x and y can assume all integers such that $0 \leq x \leq 2$ and $0 \leq y \leq 3$ and $f(x,y) = 0$ otherwise.

a) Find the value of the constant c (5)

b) Find $P(X = 2, Y = 1)$ (2)

c) Calculate Var(X) and Var(Y) (6)

d) Find Cov (X,Y) (4)

e) Find correlation of X and Y (3)

B10. Let X be a random variable defined on a sample space $S = (0,1,2, \ldots)$ with probability mass function $p(x) = \begin{cases} p(1-p)^x & x \in S,\ 0 < p < 4 \\ 0 & otherwise \end{cases}$

Let $A = \{X \geq n\}$, $B = \{X > m\}$ and $C = \{X > m + n\}$

a) Show that $P(X = x)$ is a probability mass function on S. (2)

b) Is X a continuous variable or discrete variable? Explain your choice. (2)

c) Find the probability generating function of X. (4)

d) State Baye's Theorem on conditional probability. (3)

e) Use theorem on conditional probability to show that $P(C/B) = P(A)$. (9)

END OF EXAMINATION PAPER