NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION

'ARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION

CALCULUS PST 1331

Special Examination Paper

August 2024

This examination paper consists of 4 pages

Time Allowed: **3 hours**

Total Marks: 100

Special Requirements: None

Examiner's Name: Mrs K Moyo

INSTRUCTIONS

1. Answer **ALL** questions in section A and **THREE** questions in section B
2. Marks will be allocated as indicated. Each question should start on a fresh page.

MARK ALLOCATION

QUESTION	MARKS
A1	9
A2	13
A3	10
A4	8
B5	20
B6	20
B7	20
B8	20
TOTAL	100

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A4.

A1 Solve the following inequalities

(a) $|x-3|<5$ [2]
(b) $(x-2)(x+3)>0$ [3]
(c) $|3x-2| \leq |x-1|$ [4]

A2. (a) Find the domain and the range of the function f defined by

$$f(x)=\begin{cases} x+1 & if\ -1<x<0 \\ x & if\ 0 \leq x \leq 1 \end{cases}$$ [4]

(b) Find the limits of the following functions

(i) $\lim_{x\to 2} \sqrt{x^2+5}$ [2]

(ii) $\lim_{y\to 2} (y^2-\frac{1}{y})$ [3]

(iii) $\lim_{x\to 1} \frac{x^3-1}{x-1}$ [4]

A3. Differentiate from first principles the following functions:

(a) $f(x)=3x+5$ [3]
(b) $f(x)=sinx$ [3]
(c) $f(x)=2x^3+3x-1$ [4]

A4. Suppose f and g are given by $f(x)=6$ and $g(x)=\frac{3}{7-2x}$ find

(a) $(gof)(x)$ [2]
(b) $g^{-1}(x)$ [3]
(c) $g(g^{-1}(g(x)))$ [3]

SECTION B (60 marks)

Candidates may attempt THREE questions being careful to number them B5 to B8.

B5. a) Show that $\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^x = e^a$ [4]

b) Given that $f(x) = 2x^2 - 5x + 6$, use δ, ϵmethod to prove that $\lim_{x\to 2} f(x) = 4$ [5]

c) Evaluate $\lim_{x\to\infty}\left(\frac{2x-3}{3x-7}\right)^4$ [4]

(d) Find the absolute maximum and minimum of each function on the given interval:

(i) $f(x) = 2x^3 - 5x^2 + 4x - 1$ on [-1,2] [3]

(ii) $f(x) = \frac{x^2+3}{x+1}$ on [0,3] [4]

B6. (a) Consider the curve $f(x) = 3x^2 - xy + 4y^2 = 141$

(i) Find a formula in x and y for the slope of the tangent line at any point (x,y) of the curve [5]

(ii) Write the slope-intercept equation of the line tangent to the curve at the point (1,6) [3]

(iii) Find the coordinates of all other points on the curve where the slope of the tangent line is the same as the slope of the tangent line at (1,6). [4]

(b) Evaluate the following integrals:

(i) $\int_0^1 (x^2 + 2x)dx$ [4]

(ii) $\int_a^{a+2\pi} sindx$ [4]

B7 a) State the mean value theorem. [2]

(b) Verify the mean-value theorem for $f(x) = x^3 - 6x^2 - 4x + 30$ on the interval [4, 6]. [4]

(c) Show that $f(x) = 2x^3 + x - 4 = 0$ has exactly one real solution. [4]

(d) Prove by induction that

(i) $\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1) for\ all\ n \in Z^+$ [5]

(ii) Given that A=$\begin{pmatrix}1 & 2\\ 0 & 3\end{pmatrix}$ then $A^n = \begin{pmatrix}1 & 3^n-1\\ 0 & 3^n\end{pmatrix}$ for n=1, 2, 3... [5]

B8 a) Show that $\frac{d^n}{dx^n}(sinx) = \sin\left(x+\frac{1}{2}n\pi\right)$ [5]

b) Given that$f(x) = x^2 - 5x - 6$, find the point c on (-1,6) such that $f'(c) = 0$ [4]

c) Find the integral $\int \frac{3x^3+11x^2+3x+2}{x(x+1)^3}dx$ and hence find

$$\int \frac{4x^5+19x^4+36x^3+36x^2+10x+2}{x(x+1)^3}dx$$ [11]