



NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION

DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION

BACHELOR OF EDUCATION-MATHEMATICS

CALCULUS 2 PST 1331

Main Examination Paper

November, 2024

This examination paper consists of 3 pages

Time Allowed: 3 hours

Total Marks: 100

Special Requirements: None

Examiner's Name: Mrs K Moyo

External Examiner: Dr Sunzuma G

INSTRUCTIONS

1. Answer **ALL** questions in section A and **THREE** questions in section B
2. Marks will be allocated as indicated. Each question should start on a fresh page.

MARK ALLOCATION

QUESTION	MARKS
A1	10
A2	10
A3	10
A4	10
B5	20
B6	20
B7	20
B8	20
TOTAL	100

SECTION A [40 marks]

Attempt all the questions in this section

A1 Integrate the following functions

(a) $\int e^x \sin x dx$ [5]

(b) $\int x^3 e^{x^2} dx.$ [5]

A2. (a) Use the $\epsilon - \delta$ definition to prove that $\lim_{x \rightarrow 2} 4x - 5 = 3$ [5]

(b) Evaluate $\lim_{(x,y) \rightarrow (2,1)} (2x + 5xy - 3y^2)$ [5]

A3. (a) Find the partial derivatives f_x and f_y of

$f(x, y) = 4x^3 - 3x^2y^2 + 2x + 3y$ [4]

(b) If $x^2 - xy + y^2 = 3$, find y' and y'' . [6]

A4 (a) Find the maximum of $f(x, y) = 9 - x^2 - y^2$ subject to $x + y = 3$
[10]

SECTION B [60 marks]

Answer three (3) questions from this section

B5 (a) Let $f(x, y, z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$ show that $f_{xx} + f_{yy} + f_{zz} = 0$.
[10]

(b) If $z = u^2 - v^3$ and $u = e^{2x-3y}$, $v = \sin(x^2 - y^2)$. Find z_x and z_y .
[10]

B6 (a) Let $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 6x + 8$. Find the points at which f has a relative maximum or minimum and the intervals at which f is increasing or decreasing.
[10]

(b) Let $z = f(x, y) = x + 3xy^2 + 5x^3y$. Use the definition of a derivative to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$
[10]

B7 (a) Sketch the region over which $\int_0^{2a} \int_{\sqrt{ax}}^{\sqrt{6ax-x^2}} 2xydydx$ ($a > 0$) is taken and evaluate it.
[10]

(b) Find the absolute maximum and minimum of the following function on the given interval $f(x) = \frac{x^2+3}{x+1}$ on $[0,3]$.
[10]

B8 (a) If $f(x, y, z) = xy^2 + 3x^2 - z^3$. Find $\nabla f(x, y, z)$ at $(2, -1, 4)$. [5]

(b) Prove that if $u = g(x, y)$, $v = h$, $x = k(s, t)$ and $y = r(s, t)$ then

$$\frac{\partial(u,v)}{\partial(s,t)} = \frac{\partial(u,v)}{\partial(x,y)} \cdot \frac{\partial(x,y)}{\partial(s,t)}$$

[10]

(c) Find the value of x_0 prescribed by the law of the mean when $f(x) = 3x^2 + 4x - 3$

and $a=1; b=3.$

[5]

END OF PAPER