



NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION

**DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY
EDUCATION**

CALCULUS 2 PST 1331

Examination Paper

December 2024

This examination paper consists of 3 pages

Time Allowed: 3 hours

Total Marks: 100

Special Requirements: None

Examiner's Name: Mrs K Moyo

INSTRUCTIONS

1. Answer **ALL** questions in section A and **THREE** questions in section B
2. Marks will be allocated as indicated. Each question should start on a fresh page.

MARK ALLOCATION

QUESTION	MARKS
A1	10
A2	10
A3	10
A4	10
B5	20
B6	20
B7	20
B8	20
TOTAL	100

SECTION A [40 marks]

Attempt all the questions in this section

A1 Solve the following inequalities:

(a) $x^2 < 3x + 10$ [5]

(b) $\left| \frac{2}{x} - 3 \right| < 5$. [5]

A2 Find the domain and range of the following functions:

a) $f(x) = \begin{cases} x + 2 & \text{if } -1 < x < 0 \\ x & \text{if } 0 \leq x < 1 \end{cases}$. [4]

b) $g(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{if } x \neq 2 \\ 4 & \text{if } x = 2 \end{cases}$. [6]

A3 a) Given (a) $f(x) = x^2 - 3x$, find $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$. [5]

(b) $f(x) = \sqrt{5x + 1}$, find $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ when $x > -\frac{1}{5}$. [5]

A4 (a) State the intermediate value theorem. [2]

(b) Find the absolute maximum and minimum of $f(x) = x^3 - x^2 - x + 2$ on $[0, 2]$. [8]

SECTION B [60 marks]

Answer three (3) questions from this section

B5 a) Let $u = x^3y^2 - 3xy^4 + xz^4 - 2x^2z^3$. Obtain the total differential of u . Evaluate $u = (1, 1, -1)$ and find $u(1+10^{-3}, 1 + 10^{-3}, -(1 + 10^{-3}))$ correct to 4 significant figures. Obtain, as a function of x, y, z : $\frac{\partial^5 u}{\partial x^3 \partial y^2}$ and $\frac{\partial^4 u}{\partial x \partial z^3}$. [10]

b) Given $x^3y + xy^3 = 2$, find y' and y'' at the point $[1, 1]$. [10]

B6 a) Let $y = x \sin x$. Find y''' . [10]

b) Find the directional derivative of $f(x, y) = 2x^2y^3 + 6xy$ at $(1, 1)$ in the direction of the unit vector whose angle with the positive x -axis is $\frac{\pi}{6}$. [5]

c) Find the maximum of $f(x, y) = 9 - x^2 - y^2$ subject to $x + y = 3$. [5]

B7 a) Evaluate the integral $\int_0^3 \int_0^4 \int_{x=\frac{y}{2}}^{\frac{y}{2}+1} \left(\frac{2x-y}{2} + \frac{z}{3} \right) dx dy dz$ by applying the transformations

$u = \frac{2x-y}{2}, v = \frac{y}{2}$ and $w = \frac{z}{3}$ and integrating over an appropriate region. [10]

b) Find the volume of the 3-dimensional region enclosed by the surfaces

$$\begin{aligned} z &= x^2 + 3y^2 \\ z &= 8 - x^2 - y^2. \end{aligned} \quad [10]$$

B8 a) Evaluate the following repeated integral by first reversing the order of integration:

$$\int_{x=0}^8 \int_{y=x^{\frac{2}{3}}}^2 x \cos y^4 dy dx \quad [10]$$

b) Evaluate $\int_0^{2\pi} \int_{\frac{\theta}{2\pi}}^{\frac{\theta}{2\pi}} \int_0^{3+24r^2} dz dr d\theta$. [10]

End of paper