NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION

DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION

BACHELOR OF SCIENCE EDUCATION (HONOURS) Degree in mathematics

VECTOR CALCULUS (PST 2030)

Special Examination Paper **AUGUST 2024**

This Examination Paper consists of 5 printed pages

Time Allowed: 3 hours

Total Marks: 100

Special Requirements: None

Internal Examiner: Mr Mkwelie N

INSTRUCTIONS

1. Candidates should attempt **All** questions in Section A and any other **THREE** questions in section B.
2. Each question should start on a fresh page.
3. Section A carries 40 marks.
4. Each question in Section B carries 20 marks.
5. Omission of essential working will result in loss of marks.
6. Candidates will be penalised for use of wrong mathematical notation.

MARK ALLOCATION

QUESTION	MARKS
A1-6	40
B7	20
B8	20
B9	20
TOTAL	100

Section A [40 Marks]

Candidates should attempt ALL questions, being careful to number them **A1** to **A5.**

A1. Let $\vec{A} = 2\hat{\imath} + 4\hat{\jmath} + 6\hat{k}$ and $\vec{B} = \hat{\imath} - 3\hat{\jmath} + 2\hat{k}$

a) Evaluate

(i) $\vec{A}.\vec{B}$ [2]

(ii) $\vec{A} \times \vec{B}$ [3]

(iii) Calculate the angle between vectors $\vec{A}$ *and* $\vec{B}$ [3]

(b) Find the volume of a parallelepiped whose edges are represented by $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $\vec{b} = 4\hat{\imath} + 5\hat{\jmath} + 6\hat{k}$ and $\vec{c} = 7\hat{\imath} + 8\hat{\jmath} + 6\hat{k}$. [3]

A2. (a) Show that $\vec{A} = \frac{1}{\sqrt{2}}\vec{\imath} + \frac{1}{\sqrt{2}}\vec{\jmath} +$ is a unit vector. [3]

(b) Evaluate

(i) $\lim_{t\to 0}\left(e^t\hat{\imath} + \frac{1}{t}\hat{\jmath} + 3t^2\hat{k}\right)$ [3]

(ii) $\int\left(e^t\hat{\imath} + \hat{\jmath} + e^{-t}\hat{k}\right)dt$ [3]

A3. Find the area of a ΔPQR with vertices P(1;4;6) , $Q(-2; 5; -1)$ and R(1;−1;1) [3]

A4. (a) If $\vec{a}$ and $\vec{b}$ are two vectors on a plane, give a geometrical interpretation of the vector products.

(i) $\vec{a} \times \vec{b}$. [2]

(ii) $|\vec{a} \times \vec{b}|$ [2]

(b) Given that $\phi(x, y, z) = 3xy^3 - y^2z^2$.

Find $\nabla\phi$(or $grad\phi$) at the point $P(1, 1, 2)$. [3]

A5. Let $\vec{\boldsymbol{F}} = (2xy + z^3)\hat{\imath} + x^2\hat{\jmath} + 3xz^2\hat{k}$.

a) Show that $\vec{F}$ is conservative force field. [4]

b) Find the scalar potential $\phi(x, y, z)$. [4]

c) Hence, find the work done in moving an object from $(1, -2, 1)$ and $(3, 1, 4)$. [2]

SECTION B (60 marks)

Answer any THREE questions being careful to number them B6 to B9.

B6. a) Find the scalar triple product of the vectors $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\vec{c} = 6\hat{\imath} - 8\hat{\jmath} + 10\hat{k}$. Interpret your answer. [3]

b) Given that $\vec{F} = e^{xy}\hat{\imath} + (x-y)\hat{\jmath} + x(\sin y)\,\hat{k}$, Find

(i) $\vec{F}_x$ [3]

(ii) $\vec{F}_y$ [3]

(iii) $\vec{F}_x \times \vec{F}_y$ [4]

c(i) Let $\vec{F}(x,y,z)$ be a vector valued function.State the definition of a partial derivative of $\vec{F}(t)$ with respect to x ie $\vec{F}_x$. [3]

(ii)Hence find the derivative of a vector fuction $\vec{F}(t) = t^2\hat{\imath} + 2t\hat{\jmath}$ [4]

B7. a) A particle moves along the curve $r(t) = t^2\hat{\imath} + t^3\hat{\jmath}$.

Find,

(i) the unit tangent vector, $\boldsymbol{T(t)}$. [4]

(ii) the value of $\boldsymbol{T}(2)$. [3]

(iii) principal normal vector, **N(t)** [4]

(b)Evaluate the integral $\oint_C \boldsymbol{F.dr}$,

where $\vec{F} = (3x^2 + 6y)\hat{\imath} - 14yz\hat{\jmath} + 20x^2\hat{k}$, from $(0,0,0)$ *to* $(1,1,1)$ along the curve given parametrically by $x = t, y = t^2, z = t^3$. [6]

B8. a) Determine the unit vector to the plane of

$\vec{A} = 2\hat{\imath} - 6\hat{\jmath} - 3\hat{k}$ and $\vec{B} = 4\hat{\imath} + 3\hat{\jmath} - \hat{k}$. [3]

b) A particle moves along the curve represented by $r(t) = t^2\hat{\imath} + t^3\hat{\jmath}$.

Find,

(i) the unit tangent vector, $\boldsymbol{T(t)}$. [3]

(ii) the value of $\boldsymbol{T}(2)$. [3]

(iii) the principal normal vector, **N(t)** [4]

(iv)the torsion. [3]

c) Given that $\phi(x,y,z) = xy^2z$. and $\vec{\boldsymbol{A}} = xz\hat{\imath} - xy^2\hat{\jmath} + yz^2\hat{k}$.

Find $\frac{\partial^2}{\partial y \partial z}(\phi \mathbf{A})$ at the point $P(2, -1, 1)$. [4]

B9. a) Given that $\vec{A} = x^2z^2\hat{\imath} + 2y^2z^2\hat{\imath} + xy^2z\hat{k}$. Find $curl\,\vec{A}\,(\nabla \times \vec{A})$ at point $P((1, -1, 1)$. [4]

b) Given that $\vec{A} = (2x^2y - x^4)\hat{\imath} + (e^{xy} - y\sin x)\hat{\jmath} + xy^2z\hat{k}$Find $\frac{\partial A}{\partial x}$ and $\frac{\partial A}{\partial y}$ [6]

(c) Given that $\phi(x, y, z) = 6x^3y^3z$.

(i) Find $\nabla.\nabla\phi$(or div gradϕ). [3]

(ii) Show that $\nabla.\nabla\phi = \nabla^2\,\phi$ [3]

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ denotes the Laplacian operator. [4]

B10. a) State Green's Theorem. [3]

b) Verify Green's theorem in the plane for $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the closed region bounded by $y = x\ and\ y = x^2$ [12]

c))A particle which moves around a circle at a constant rate satisfies

$\hat{r} = a\cos\omega t\hat{\imath} + a\sin\omega t\hat{\jmath}$ where a and ω are constants.

a) Show that

(i).velocity $\vec{v}(t)$ is perpendicular to $\hat{r}$. [3]

(ii).acceleration $\vec{a}(t)$is given by $-\omega^2\hat{r}$. [2]

END OF EXAMINATION PAPER