NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION

DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION

BACHELOR OF SCIENCE EDUCATION (HONS) DEGREE IN MATHEMATICS

VECTOR CALCULUS (PST 2030)

Main Examination Paper **November 2024**

This Examination Paper consists of 4 printed pages

Time Allowed: 3 hours

Total Marks: 100

Special Requirements: None

Internal Examiner: Mr Mkwelie N

External Examiner: Dr Sunzuma G

INSTRUCTIONS

1. Candidates should attempt **All** questions in Section A and any other **THREE** questions in section B.
2. Each question should start on a fresh page.
3. Candidates will be penalised for use of wrong mathematical notation.

MARK ALLOCATION

QUESTION	MARKS
A1-5	40
B6	20
B7	20
B8	20
B9	20
TOTAL	100

Section A [40 Marks]

Candidates should attempt ALL questions, being careful to number them **A1** to **A5.**

A1. The three points A, B and C have position vectors

$3\hat{\imath} - \hat{\jmath} - 2\hat{k},\ 7\hat{\imath} + 2\hat{\jmath} + 7\hat{k}$ and $\hat{\imath} + \hat{\jmath} + 3\hat{k}$ relative to the origin.

Find,

a) the Cartesian equation of the line AB. [5]

b) the vector equation of plane ABC. [5]

c) the volume of the parallelepiped whose edges are OA; OB and OC. [5]

A2. (a) Given that $\vec{u} = a\hat{\imath} - 2\hat{\jmath} + \hat{k}$ and $\vec{u} = 2a\hat{\imath} + a\hat{\jmath} - 4\hat{k}$ Find the values of a so that $\vec{u}$ and $\vec{v}$ are perpendicular. [4]

(b) Evaluate

(i) $\lim_{t\to 0}\left((t^2+1)\hat{\imath} + \cos t\,\hat{\jmath} + e^{-t}\hat{k}\right)$ [3]

(ii) $\int\left(3t^2\hat{\imath} + \sin t\,\hat{\jmath} + \frac{1}{t}\hat{k}\right)dt$ [3]

A3. Show that $\vec{A} = (cos\theta; sin\theta cos\phi; sin\theta sin\phi)$ is a unit vector. [3]

A4. (a) A particle moves along the space curve $\vec{r}(t) = e^{-t}\cos t\,\hat{\imath} + e^{-t}\sin t\,\hat{\jmath} + e^{-t}\hat{k}$.

Calculate,

(i) the magnitude of the velocity, [4]

(ii) the magnitude of the acceleration, [4]

A5. Determine the value of α if the vector field

$\vec{F}(x,y,z) = (-4x - 6y + 3z)\hat{\imath} + (-2x + y - 5z)\hat{\jmath} + (5x + 6y + \alpha z)\,\hat{k}$ is solenoidal. [4]

SECTION B (60 marks)

Answer any **THREE** questions being careful to number them B6 to B9.

B6. (a) Given that $\vec{F} = e^{xy}\hat{\imath} + (x-y)\hat{\jmath} + x(\sin y)\,\hat{k}$, Find

(i) $\vec{F}_x$ [3]

(ii) $\vec{F}_{xx}$ [3]

(iii)$\vec{F}_{xy}$ at the point $P(0,0,0)$ [4]

(b) Determine the constants α and β such that the vector field

$\vec{\mathrm{F}} = (x+\alpha y)\hat{\imath} + (y+\beta x)\hat{\jmath} + z\,\hat{k}$ is conservative. [5]

(a) Given that $r'(t) = \cos t\,\hat{\imath} - \sin t\,\hat{\jmath} + \frac{1}{1+t^2}\,\hat{k}$ and $\vec{r}(0) = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$.

Find $r(t)$. [5]

B7. Verify the Greens theorem in the plane for

$$\oint_c (4x + 2y^2)\,dx + (4xy + e^y)dy$$

where C is the boundary of the region between $y = x^2$ *and* $y = \sqrt{x}$. [10]

(b) Given that $\vec{\mathbf{F}} = (3x^2 + 6y)\hat{\imath} - 14yz\hat{\jmath} + 20xz^2\hat{k}$, evaluate the line integral $\oint_C \boldsymbol{F}.\boldsymbol{dr}$, from $(0,0,0)$ *to* $(1,1,1)$ along the curve given parametrically by

$x = t, y = t^2, z = t^3$. [10]

B8. Let C be the curve defined by $r(t) = a\cos t\,\hat{\imath} + a\sin t\,\hat{\jmath} + bt\hat{k}$.

Find,

(a) the unit Tangent vector, ***T*** [4]

(b) the Principal normal vector, N [5]

(c) the Binormal vector B, [6]

(d) the curvature, κ. [3]

(e) the torsion τ. [2]

B9. a) Given that $\vec{A} = x^2z^2\hat{\imath} - 2y^2z^2\hat{\imath} + xy^2z\hat{k}$. At point P ((1, −1, 1). Find

(i) $\nabla.\vec{A}\ (div\ \vec{A})$. [5]

(ii) $\nabla \times \vec{A}(\text{curl } \vec{A})$. [5]

(b) Given that $\vec{F} = (-4x - 3y + az)\hat{\imath} + (bx + 3y + 5z)\hat{\imath} + (4x + cy - 3z)\hat{k}$.

Find the constants $a, b, and\ c$ if $\vec{F}$ is solenoidal. [6]

(c) Evaluate $\lim_{t\to\infty}\left(e^{-t}\hat{\imath} + \frac{1}{t}\hat{\jmath} + \frac{t}{t^2+1}\hat{k}\right)$ [4]

END OF EXAMINATION PAPER