NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION

DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION

BACHELOR OF SCIENCE EDUCATION (HONS) DEGREE IN MATHEMATICS

VECTOR CALCULUS (PST 2030)

Examination Paper **December 2024**

This Examination Paper consists of 4 printed pages

Time Allowed: 3 hours

Total Marks: 100

Special Requirements: None

Internal Examiner: Mr Mkwelie N

External Examiner: Dr Sunzuma G

INSTRUCTIONS

1. Candidates should attempt **All** questions in Section A and any other **THREE** questions in section B.
2. Each question should start on a fresh page.
3. Candidates will be penalised for use of wrong mathematical notation.

MARK ALLOCATION

QUESTION	MARKS
A1-5	40
B6	20
B7	20
B8	20
B9	20
TOTAL	100

Section A [40 Marks]

Candidates should attempt ALL questions, being careful to number them **A1** to **A5.**

A1. Let $\vec{A} = 2\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $\vec{B} = 7\hat{\imath} + 24\hat{k}$

a) Evaluate

(i) $\vec{A}.\vec{B}$ [2]

(ii) $\vec{A} \times \vec{B}$ [4]

(iii) Calculate the angle between vectors $\vec{A}$ and $\vec{B}$ [3]

(b) Find the volume of the parallelepiped whose edges are OA; OB; OC when A (1; 2; 3); B (1; 1; 2); C (2; 1; 1) [4]

A2. (a) Given that $\vec{u} = a\hat{\imath} - 2\hat{\jmath} + \hat{k}$ and $\vec{u} = 2a\hat{\imath} + a\hat{\jmath} - 4\hat{k}$ Find the values of a so that $\vec{u}$ and $\vec{v}$ are perpendicular. [4]

(b) Evaluate

(i) $\lim_{t\to 0}\left((t^2+1)\hat{\imath} + \frac{\sin t}{t}\hat{\jmath} + e^{-t}\hat{k}\right)$ [3]

(ii) $\lim_{t\to\infty}\left(e^{-t}\hat{\imath} + \frac{1}{t}\hat{\jmath} + \frac{t}{t^2+2}\hat{k}\right)$ [3]

A3. Given that $r'(t) = \cos 2t\,\hat{\imath} - 2\sin t\,\hat{\jmath} + \frac{t}{1+t^2}\,\hat{k}$ and $\vec{r}(0) = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$.

Find $r(t)$. [5]

A4. Let $\vec{F} = (2xy + z^3)\hat{\imath} + x^2\hat{\jmath} + 3xz^2\hat{k}$.

a) Show that $\vec{F}$ is conservative force field. [3]

b) Find the scalar potential $\phi(x, y, z)$. [3]

c) Hence, find the work done in moving an object from $(1, -2, 1)$ and $(3, 1, 4)$. [2]

A5. Given that $\phi(x, y, z) = 3x^2y - y^2z^2$. Find $\nabla\phi(grad\phi)$ at the point $P(1, -2, -1)$ [4]

SECTION B (60 marks)

Answer any THREE questions being careful to number them B6 to B9.

B6. a) Find the scalar triple product of the vectors $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\vec{c} = 6\hat{\imath} - 8\hat{\jmath} + 10\hat{k}$. Interpret your answer. [4]

b) Given that $\vec{F} = (x^2 + 2y)\hat{\imath} + \sin y\,\hat{\jmath} + e^{xy}\,\hat{k}$, Find

(i) $\vec{F}_x$ [3]

(ii) $\vec{F}_{xy}$ [3]

(iii) $\vec{F}_x \times \vec{F}_y$ [4]

(c) Given that $\phi(x, y, z) = 6x^3y^3z$.

(i) Find $\nabla.\nabla\phi$(or div gradϕ). [3]

(ii) Show that $\nabla.\nabla\phi = \nabla^2\,\phi$ [3]

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ denotes the Laplacian operator.

B7. (a) Verify Green's theorem in the plane for $\oint_C (4x + 2y^2)dx + (4xy + e^y)dy$ where C is the boundary of the region $y = x^2$ and $y = \sqrt{x}$. [12]

(b) Compute the line integral $\oint_C \boldsymbol{F}.\boldsymbol{dr}$, where $\vec{\mathbf{F}} = (x^2 + y^2)\hat{\imath} - 3xy\hat{\jmath} + y^3\hat{k}$,

from $(0, 0, 0)$ to $(1, 1, 1)$ along the curve given parametrically by

$x = t, y = t^2, z = t^3$. [8]

B8. Let C be the curve defined by $r(t) = a\cos t\,\hat{\imath} + a\sin t\,\hat{\jmath} + bt\hat{k}$.

Find,

(a) the unit tangent vector, ***T*** [4]

(b) the principal normal vector, N [5]

(c) the binormal vector B, the [6]

(d) the curvature, κ. [3]

(e) the torsion τ. [2]

B9. a) Given that $\vec{A} = x^2z^2\hat{\imath} + 2y^2z^2\hat{\imath} + xy^2z\hat{k}$. At point P((1, −1, 1). Find

(i) $\nabla . \vec{A} \left(div\, \vec{A} \right)$. [5]

(ii) $\nabla \times \vec{A}$(curl $\vec{A}$). [5]

(b) Given that $\vec{F} = (-4x - 3y + az)\hat{\imath} + (bx + 3y + 5z)\hat{\imath} + (4x + cy - 3z)\hat{k}$.

(i) Find the constants $a, b, and\ c$ if $\vec{F}$ is solenoidal. [4]

(ii) Express $\vec{F}$ as a gradient of a scalar function. [6]

END OF EXAMINATION PAPER