NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION

DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION

ANALYSIS (PST2031)

Main Examination Paper   **November 2024**

This Examination Paper consists of 3 printed pages

Time Allowed:	3 hours
Total Marks:	100
Special Requirements:	None
Internal Examiner:	Mr T Moyo
External Examiner:	Dr G Sunzuma

INSTRUCTIONS

1. Answer all questions in section A and any **THREE** questions in section B.

2. Each question should start on a fresh page

3. Marks will be allocated as indicated below

MARK ALLOCATION

QUESTION		MARKS
A1		12
A2		8
A3		8
A4		12
B5		20
B6		20
B7		20
B8		20
TOTAL		**100**

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SECTION A (40 MARKS)

Answer all questions in this section

A1. a) What is an axiom? (2)

b) List the six field axioms. (6)

c) State the completeness axiom. (4)

A2. a) State, without proof, the Bolzano-Weierstrass theorem. (2)

b) Define a Cauchy sequence. (2)

c) Prove that a Cauchy sequence is convergent. (4)

A3. Test for convergence or divergence of the series

a) $\sum_{k=1}^{\infty} \frac{\ln(k+2)}{k}$. (4)

b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)}}$. (4)

A4. a) Prove that a convergent sequence can have at most one limit. (4)

b) Show that a sequence of real numbers has only one limit. (4)

c) Find the derivative of $f(x) = x^2$ using definition of a derivative. (4)

SECTION B (60 marks)

Answer any THREE questions in this section

B5. a) Show that if $a < b$ and $c < 0$ then $ac > bc$. (5)

b) Show that $\lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^n = e^x$ (5)

c) Use the integral test to show that $\sum_{n=0}^{\infty} \frac{1}{n^2+1} = \frac{\pi}{2}$ (5)

d) Use telescopic method to evaluate $\sum_{n=4}^{\infty} \frac{1}{(n-3)(n+2)}$ (5)

B6. a) (i) State 2 properties of greatest lower bound (2)

(ii) Hence prove that $g.l.b.(A + B) = g.l.b.(A) + g.l.b.(B)$ (4)

b) Using the $\epsilon - \delta$ definition of continuity to show that the function $f(x) = 3x - 5$ is continuous at the point $x = 4$. (6)

c) Find the radius and interval of convergence of $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n}$ (8)

B7. a) State the Cauchy-Schwarz inequality. (2)

b) Prove the Triangle inequality. (4)

c) For what values of x is the series $\sum_{n=0}^{\infty} n! \, x^n$ converging (4)

d) Find the Maclaurin series expansion for $Sin(x)$ hence determine the interval of convergence (10)

B8. a) (i) Define a monotonic sequence. (2)

(ii) Show that the following sequence is monotonic $\left(1 + \frac{1}{n}\right)^n$. (5)

b) Determine the interval and radius of convergence of the series $\sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n(n+2)}$ (6)

c) If $x^3 - 3 = x^2 + 3x$ show that there is $c \in \mathbb{R}$ such that $f(c) = 0$ in the interval $[2; 3]$. (7)

END OF EXAMINATION PAPER