NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY
FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION
DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION

ANALYSIS (PST2031)

Examination Paper
DECEMBER 2024

This Examination Paper consists of 3 printed pages

Time Allowed:	3 hours
Total Marks:	100
Special Requirements:	None
Internal Examiner:	Mr T Moyo
External Examiner:	Dr G Sunzuma

INSTRUCTIONS

1. Answer all questions in section A and any three questions in section B.
2. Each question should start on a fresh page
3. Marks will be allocated as indicated below

MARK ALLOCATION

QUESTION	MARKS
A1	7
A2	8
A3	9
A4	16
B5	20
B6	20
B7	20
B8	20
TOTAL	**100**

Section A: Answer all questions in this section (40 marks)

A1. a) Show that the additive identity is only 0 (zero). (3)

b) Prove that $a.(-b) = -(a.b)$ (4)

A2. a) Define a subsequence? (2)

b) Define a monotonic sequence. (2)

c) Verify if the sequence $u_n = 3n - 2$ is monotonic. (4)

A3. a) Test for convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{1+cosn}{n^2}$ (4)

b) Investigate the convergence or divergence of the alternating series $\sum_{k=1}^{\infty} \frac{(-1)^k(k+3)}{(k)(k+1)}$ (5)

A4. a) State the Cauchy-Schwarz inequality (2)

b) Show that a sequence of real numbers has only one limit. (4)

c) Find the derivative of $f(x) = 2x^2 + 1$ using definition of a derivative. (5)

a) Use the Intermediate Value Theorem to show that there is a root of the equation $x^3 - 3x + 1 = 0$ in the interval $(0,1)$ (5)

SECTION B: Answer any three questions in this section (60 marks).

B5. a) Use telescopic method to evaluate $\sum_{n=1}^{\infty} \frac{1}{(n+3)(n+1)}$ (5)

b) Show that if $a < b$ and $c > 0$ then $ac < bc$. (5)

c) Show that the function $f(x) = 5x - 2$ is continuous at the point $x = 3$. (5)

d) Use the integral test to show that the series $\sum_{n=0}^{\infty} \frac{1}{n^2+2}$ converges. (5)

B6. a) Determine the limit $\lim_{n\to\infty} \left(1 - \frac{5}{n}\right)^n$ (5)

b) Prove that $l.u.b.(A + B) = l.u.b.(A) + l.u.b.(B)$ (5)

c) Verify that $f(x) = x - 6$ is Reinmann integrable on [2,4] and hence use Reinmann integration to find $\int_2^4 (x - 6)\, dx$ (10)

B7. a) State without proof the Triangle inequality. (2)

b) Prove that the sequence $u_n = \frac{1}{\sqrt{n}}$ is a null sequence (4)

c) Show that the sequence $u_n = \frac{3n-5}{7n+1}$ converges and find its limit (6)

d) Find the Maclaurin series expansion for e^x and then use it to determine the radius and interval of convergence of $f(x) = e^x$. (8)

B8. a) (i) Define a monotonic sequence. (2)

(ii) Show that the sequence is monotonic $\left(1 + \frac{1}{n}\right)^n$. (5)

b) Show that a convergent sequence is Cauchy. (5)

c) Determine the interval and radius of convergence of the series $\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$ (8)