



NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION

**DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY
EDUCATION**

QUANTUM PHYSICS 1 (PST 2170)

Main Examination Paper

NOVEMBER 2024

This Examination Paper consists of 4 printed pages

Time Allowed:	3 hours
Total Marks:	100
Special Requirements:	None Internal
Examiner:	Mrs. N. Moyo
External Examiner:	Dr N Zezekwa

INSTRUCTION

1. This paper consists of **5** questions.
2. Answer **all** questions from **section A** and any three from **section B**
3. Begin each full question on a new page.
4. Show all your working steps clearly in any calculation.

MARK ALLOCATION

QUESTION	MARKS
First question	40
Second question	20
Third question	20
Fourth question	20
TOTAL	100

FORMULAE

$$1. \int_0^{\infty} e^{-ax^2} dx = \frac{1}{2} \left(\frac{\pi}{a}\right)^{1/2}$$

$$2. \int_0^{\infty} x^2 e^{-ax^2} dx = \frac{1}{4a} \left(\frac{\pi}{a}\right)^{1/2}$$

$$3. \int_0^{\infty} x e^{-ax^2} dx = \frac{1}{2a}$$

DATA

the Planck constant

$$h = 6.63 \times 10^{-34} \text{ J s}$$

unified atomic mass unit

$$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$$

rest mass of electron

$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

rest mass of proton

$$m_p = 1.67 \times 10^{-27} \text{ kg}$$

SECTION A

1. (a) (i) What are the three reasons led to the birth of quantum mechanics? [3]
(ii) What were the shortcomings of quantum mechanics that rose as it tried to explain classical problems in (i) above. [2]
- (b) (i) List three properties of a wave function. [3]
(ii) Describe the Copenhagen and the Born interpretations of the wave function. [4]

(c) A wave function of a particle with mass m is given by the following expression:

$$\psi(x) = \begin{cases} A \cos \alpha x & -\frac{\pi}{2\alpha} \leq x \leq +\frac{\pi}{2\alpha} \\ 0 & \text{otherwise} \end{cases}, \text{ where } \alpha = 1.00 \times 10^{10} / m.$$

- (i) Find the normalization constant. [3]
- (ii) Find the probability that the particle can be found on the interval $0 < x < 0.5 \times 10^{-10} m$. [3]
- (iii) Find the particle's average position. [3]
- (iv) Find its average momentum. [3]
- (e) Find its average kinetic energy for the region: $-0.5 \times 10^{-10} m < x < +0.5 \times 10^{-10} m$. [4]

- (d) (i) The position of an electron is measured with an accuracy of 10^{-6} m. Find the uncertainty in the electron's position after 1s. Comment on this result. [3]
- (ii) Suppose this electron moves with velocity of $1,1 \times 10^{-6}$ m/s, its velocity being measured to a precision of 0.1 percent, what is the maximum precision with which its momentum could be simultaneously measured? [3]
- (e) (ii) Write down the expressions of Pauli spin matrices $\hat{\sigma}_x$, $\hat{\sigma}_y$, $\hat{\sigma}_z$, hence using prove that matrices that the spin matrices satisfy the condition
- $$\partial_x \partial_y + \partial_y \partial_x = 0 \quad [6]$$

SECTION B

2. (a) (i) Is angular momentum a scalar or vector justify your answer? [2]
- (ii) Write down the three dimensions of angular momentum, L. [6]
- (b) Show that the z, component of angular momentum, $\hat{L}_z = i\hbar \frac{\partial}{\partial \phi}$ in spherical co-ordinates. [5]
- (c) Demonstrate that, the eigenvalues of the square of the angular momentum operator are given by $\hat{L}^2 = \hbar^2 l(l+1)$ where $l = 0, 1, 2, 3, \dots$ [6]
- (d) State the principle of conservation of angular momentum. [1]
3. Consider a particle whose wave function is given by
- $$\varphi_{x,t} = \text{Cos } kx - \omega t + \text{Sin } (kx - \omega t)$$
- (a) (i) What type of a solution to the Schrodinger is $\varphi_{x,t}$ [1]
- (ii) List two solutions of the Schrodinger equation from $\varphi_{x,t}$ [2]
- (iii) Show that the two solutions in (ii) satisfy the Schrodinger equation.
- where $k = \sqrt{\frac{2ME}{\hbar}}$ [5]
- (b) (i) Obtain an expression of the eigen value equation for the Hamiltonian operator and the wave function $\varphi_{x,t}$ for which $\varphi_{x,t}$ is normalized. [2]
- (ii) What will be the value of $\langle \hat{H}^2 \rangle$ and $\langle \hat{H} \rangle^2$. [4]
- (iii) Hence show that the variance, $\sigma_{\hat{H}}^2 = \langle \hat{H}^2 \rangle - \langle \hat{H} \rangle^2$ is zero [4]

(iv) What is the significance of the variance, σH^2 being zero. [2]

4. (a) Find the inner product of the following and explain the physical significance of each of the results.

(i) $[x, P_x]$ [4]

(ii) $[H, P_x]$ [4]

(b) Show that φ_m and φ_n , the two eigen functions of a Hermitian operator \hat{A} with corresponding eigen values a_m and a_n respectively are orthogonal [5]

(c) Describe the properties of operators. [2]

(d) Consider a particle of mass m which moves freely along the x axis from $-\frac{a}{2}$ to $\frac{a}{2}$ and not allowed beyond these points. Suppose the lowest energy state of the wave function of the particle is $\varphi_{(x,t)} = A \cos \frac{\pi x}{a} e^{-\frac{iEt}{\hbar}}$, Applying Schrodinger equation, show that the energy for different states can be expressed as

$$E = \frac{\pi^2 \hbar^2}{2ma^2} \quad [5]$$

5. (a) Explain the following

(i) the connection between Plank's hypothesis of energy quanta and the energies of the quantum harmonic oscillator [3]

(ii) Why a quantum oscillator can never be at rest while a classical oscillator can be Does this violate the Bohr's correspondence principle? [4]

(b) (i) Derive the Time Independent Schrodinger Equation (TISE). [6]

(ii) Demonstrate that if φ_1 and φ_2 are solutions to the Schrodinger equation, we can generalize the equation of continuity to

$$\frac{\partial(\varphi_1\varphi_2)}{\partial t} + \frac{i\hbar}{2m} (\nabla\varphi_1\nabla\varphi_2 - \varphi_2\nabla\varphi_1) = 0 \quad [7]$$

END OF PAPER